27.09.2024.

Matrice, operacije sa matricama

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}_{m\times n}$$

$A_{m\times n}$ → broj vrsta —, → broj kolona |, dimenzije

* Sabiranje i oduzimanje samo kod matrica istih dimenzija
* Množenje matrice brojem $\alpha \in \mathbb{R}$ za bilo koju matricu

1. Naći zbir i razliku matrica A i B ako

$$A = \begin{pmatrix} -1 & 0 & 5 \\ 0 & -2 & 0 \\ 3 & 4 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

Rj $A_{3\times 3}$, $B_{3\times 3}$

$$A+B = \begin{pmatrix} -1 & 0 & 5 \\ 0 & -2 & 0 \\ 3 & 4 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 8 \\ 0 & 0 & 3 \\ 3 & 4 & 1 \end{pmatrix}_{3\times 3}$$

$$A-B = \begin{pmatrix} -1 & 0 & 5 \\ 0 & -2 & 0 \\ 3 & 4 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -3 & -1 & 2 \\ 0 & -4 & -3 \\ 3 & 4 & -1 \end{pmatrix}_{3\times 3}$$

2. Date su matrice A, B, C i D. ~~Izvršiti~~ Izračunati, ukoliko je moguće

a) $3A$

b) $3A-2B$

c) $\frac{1}{2}D+B$

d) $C-2A$

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 4 \end{pmatrix}_{3\times 2} \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}_{2\times 2}$$

$$C = \begin{pmatrix} 0 & 1 \\ -2 & 3 \\ 1 & 1 \end{pmatrix}_{3\times 2} \quad D = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}_{2\times 2}$$

a) $3A = 3\begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 3\cdot 1 & 3\cdot 2 \\ 3\cdot(-1) & 3\cdot 0 \\ 3\cdot 3 & 3\cdot 4 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ -3 & 0 \\ 9 & 12 \end{pmatrix}_{3\times 2}$

b) $3A - 2B = \begin{pmatrix} 3 & 6 \\ -3 & 0 \\ 9 & 12 \end{pmatrix}_{3\times 2} - \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}_{2\times 2}$

nije moguće izvršiti oduzimanje
jer matrice nisu istog tipa (istih dimenzija)

c) $\frac{1}{2}D + B = \begin{pmatrix} \frac{1}{2} & 0 \\ 1 & -\frac{1}{2} \end{pmatrix}_{2\times 2} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}_{2\times 2} = \begin{pmatrix} \frac{3}{2} & 2 \\ 4 & \frac{7}{2} \end{pmatrix}_{2\times 2}$

d) sami

3. Naći maticu X ako je $A + 3X = 2B$, gdje je

$A = \begin{pmatrix} 2 & 0 & 3 \\ 0 & -2 & 4 \\ 0 & 3 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & -3 \\ -1 & 0 & 0 \end{pmatrix}$

Rj.
$$A + 3X = 2B$$
$$3X = 2B - A \quad /\cdot \frac{1}{3}$$
$$X = \frac{2}{3}B - \frac{1}{3}A$$
$$X = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & 2 \\ 0 & \frac{4}{3} & -2 \\ -\frac{2}{3} & 0 & 0 \end{pmatrix} - \begin{pmatrix} \frac{2}{3} & 0 & 1 \\ 0 & -\frac{2}{3} & \frac{4}{3} \\ 0 & 1 & -\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 0 & \frac{2}{3} & 1 \\ 0 & 2 & -\frac{10}{3} \\ -\frac{2}{3} & -1 & \frac{1}{3} \end{pmatrix}$$

* Množenje matrica je moguće ukoliko je broj kolona prve matrice jednak broju vrsta druge

$A_{m\times \underline{n}} \cdot B_{\underline{n}\times k} = C_{m\times k}$

4. Koje od matrica se i u kom redoslijedu mogu pomnožiti?

$$A = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}_{2\times2} \qquad B = \begin{pmatrix} -2 & 1 \\ 0 & 3 \\ 4 & 0 \end{pmatrix}_{3\times2} \qquad C = \begin{pmatrix} -3 & 0 & 1 \\ 0 & -2 & 2 \end{pmatrix}_{2\times3}$$

(Razmatrati i slučaj množenja matrice same sa sobom)

$A_{2\times\underline{2}} \cdot A_{\underline{2}\times2}$ → može, rezultat matrica reda 2 (kvadratna 2x2)

$$A \cdot A = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 4 \end{pmatrix}_{2\times2}$$

$A_{2\times\underline{2}} \cdot B_{\underline{3}\times2}$ ne može

$A_{2\times\underline{2}} \cdot C_{\underline{2}\times3}$ može, rezultat matrica 2x3

$$A \cdot C = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} -3 & 0 & 1 \\ 0 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 1 \\ 0 & -4 & 4 \end{pmatrix}_{2\times3}$$

$B_{3\times\underline{2}} \cdot A_{\underline{2}\times2}$ može, rezultat matrica 3x2

$$B \cdot A = \begin{pmatrix} -2 & 1 \\ 0 & 3 \\ 4 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 6 \\ -4 & 4 \end{pmatrix}_{3\times2}$$

$A \cdot B$ ne može, $B \cdot A$ može ⇒ množenje matrica nije komutativno!!!

$B_{3\times\underline{2}} \cdot B_{\underline{3}\times2}$ ne može

$B_{3\times\underline{2}} \cdot C_{\underline{2}\times3}$ može, rezultat matrica 3x3

$$B \cdot C = \begin{pmatrix} -2 & 1 \\ 0 & 3 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} -3 & 0 & 1 \\ 0 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 6 & -2 & 0 \\ 0 & -6 & 6 \\ -12 & 0 & 4 \end{pmatrix}_{3\times3}$$

$C \cdot A$, $C \cdot B$ i $C \cdot C$ sami

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}_{m\times n} \xrightarrow{\text{transponovanje}} A^T = \begin{pmatrix} a_{11} & \cdots & a_{m1} \\ a_{1n} & \cdots & a_{mn} \end{pmatrix}_{n\times m}$$

5. Izračunati $A^2B - 3B^T + 2AB$ za $A = \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 2 \\ 0 & -5 \end{pmatrix}$

Rj:

$$A^2 = A\cdot A = \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ -9 & 1 \end{pmatrix}$$

$$A^2B = \begin{pmatrix} 4 & 0 \\ -9 & 1 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 0 & -5 \end{pmatrix} = \begin{pmatrix} 16 & 8 \\ -36 & -23 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 4 & 0 \\ 2 & -5 \end{pmatrix} \rightarrow 3B^T = \begin{pmatrix} 12 & 0 \\ 6 & -15 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ 0 & -5 \end{pmatrix} = \begin{pmatrix} 8 & 4 \\ -12 & -11 \end{pmatrix} \rightarrow 2AB = \begin{pmatrix} 16 & 8 \\ -24 & -22 \end{pmatrix}$$

$$A^2B - 3B^T + 2AB = \begin{pmatrix} 16 & 8 \\ -36 & -23 \end{pmatrix} - \begin{pmatrix} 12 & 0 \\ 6 & -15 \end{pmatrix} + \begin{pmatrix} 16 & 8 \\ -24 & -22 \end{pmatrix} =$$

$$= \begin{pmatrix} 20 & 16 \\ -66 & -30 \end{pmatrix}$$

• Neke bitne matrice i osobine

O → nula matrica (svi elementi su 0)

E → jedinična (samo kvadratna, jedinice po dijagonali, ostalo nule)

$$E_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \cdots & 0 \\ 0 & 0 & 0 & & 1 \end{pmatrix}_{n\times n}$$

$A\cdot E = E\cdot A \quad (A_{n\times n})$

$A(B+C) = AB + AC$

$(B+C)A = BA + CA$

$A^m \cdot A^n = A^{m+n}$

$(A^m)^n = A^{mn}$

$(AB)C = A(BC)$

$(A^T)^T = A$

$(A+B)^T = A^T + B^T$

$(AB)^T = B^T A^T$

$(kA)^T = kA^T, \ k\in\mathbb{R}$

6. Pokazati da ~~gee~~ $B = \begin{pmatrix} 5 & 2 & -3 \\ 1 & 3 & -1 \\ 2 & 2 & -1 \end{pmatrix}$ anulira polinom $p(x) = x^3 - 7x^2 + 13x - 5$

Rj: $p(B) = B^3 - 7B^2 + 13B - \underline{\underline{5E}}$!

$$B^2 = B \cdot B = \begin{pmatrix} 5 & 2 & -3 \\ 1 & 3 & -1 \\ 2 & 2 & -1 \end{pmatrix} \begin{pmatrix} 5 & 2 & -3 \\ 1 & 3 & -1 \\ 2 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 21 & 10 & -14 \\ 6 & 9 & -5 \\ 10 & 8 & -7 \end{pmatrix}$$

$$B^3 = B^2 \cdot B = \begin{pmatrix} 21 & 10 & -14 \\ 6 & 9 & -5 \\ 10 & 8 & -7 \end{pmatrix} \begin{pmatrix} 5 & 2 & -3 \\ 1 & 3 & -1 \\ 2 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 87 & 44 & -59 \\ 29 & 29 & -22 \\ 24 & 30 & -31 \end{pmatrix}$$

$$p(B) = B^3 - 7B^2 + 13B - 5E = \begin{pmatrix} 87 & 44 & -59 \\ 29 & 29 & -22 \\ 24 & 30 & -31 \end{pmatrix} - \begin{pmatrix} 147 & 70 & -98 \\ 42 & 63 & -35 \\ 70 & 56 & -49 \end{pmatrix} +$$

$$+ \begin{pmatrix} 65 & 26 & -39 \\ 13 & 39 & -13 \\ 26 & 26 & -13 \end{pmatrix} + \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

7. Naći sve matrice reda 2 t.d. $A = A^T$

Rj: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad A^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$A = A^T$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \qquad \begin{array}{l} a = a \;\; T \\ \left.\begin{array}{l} b = c \\ c = b \end{array}\right\} b = c \\ d = d \end{array}$$

$$A = \begin{pmatrix} a & b \\ b & d \end{pmatrix} \quad a, b, d \in \mathbb{R}$$

Def: Matrica A je simetrična ako $A = A^T$
Matrica A je antisimetrična ako $A = -A^T$

8. Dopuniti matricu A tako da bude simetrična

$$A = \begin{pmatrix} 1 & -5 & 2 \\ \underline{-5} & 0 & 4 \\ \underline{2} & \underline{4} & -4 \end{pmatrix}$$

9. Odrediti kvadratni korijen matrice $A = \begin{pmatrix} 16 & 0 \\ 14 & 0 \end{pmatrix}$

* Korijen matrice A je matrica B t.d. $B^2 = A$

Rj.
$$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$B^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+bc & ab+bd \\ ca+dc & cb+d^2 \end{pmatrix} = \begin{pmatrix} 16 & 0 \\ 14 & 0 \end{pmatrix}$$

$$\begin{aligned} a^2+bc &= 16 \\ ab+bd &= b(a+d) = 0 \Rightarrow \boxed{b=0} \text{ ili } \cancel{\boxed{a+d=0}} \text{ nemoguće} \\ ca+dc &= c(a+d) = 14 \\ cb+d^2 &= 0 \end{aligned}$$

$$\Rightarrow \quad \begin{aligned} a^2 &= 16 \Rightarrow a = \pm 4 \\ c(a+d) &= 14 \Rightarrow c = \pm \frac{7}{2} \\ d^2 &= 0 \Leftrightarrow \boxed{d=0} \end{aligned}$$

$$(a,b,c,d) = (4, 0, \tfrac{7}{2}, 0) \text{ ili}$$
$$(a,b,c,d) = (-4, 0, -\tfrac{7}{2}, 0)$$

provjera

$$\begin{pmatrix} 4 & 0 \\ \frac{7}{2} & 0 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ \frac{7}{2} & 0 \end{pmatrix} = \begin{pmatrix} 16 & 0 \\ 14 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -4 & 0 \\ -\frac{7}{2} & 0 \end{pmatrix}\begin{pmatrix} -4 & 0 \\ -\frac{7}{2} & 0 \end{pmatrix} = \begin{pmatrix} 16 & 0 \\ 14 & 0 \end{pmatrix}$$