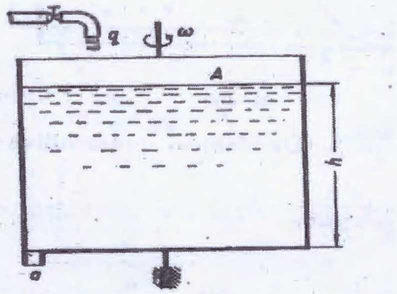
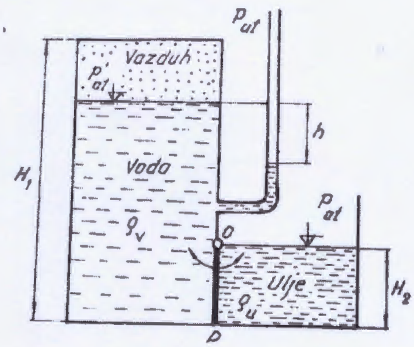
Prvi kolokvijum iz Mehanike fluida

(14.12.2020.)

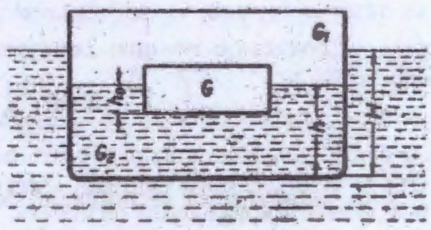
1. **(15p)** Cilindrični sud kružnog presjeka površine A napunjen tečnošću do visine h obrće se konstantnom ugaonom brzinom ω oko svoje vertikalne ose. Naći brzinu v i dotok Q pri kome nivo vode u sudu ostaje konstantan, ako je za vrijeme obrtanja otvor presjeka a na periferiji dna suda otvoren.



2. **(8p)** Zatvarač OP izmrđu dva rezervoara može se okretati bez trenja oko tačke O, prema slici. Kolikom silom i u kom smjeru treba djelovatu na zatvarač u tački P da bi bio u ravnoteži u vertikalnom položaju? Dati su podaci: H_1 = 5 m, H_2 = 2 m, h = 2 m ρ_v = 1000 kg/m^3, ρ_u = 800 kg/ m^3.



3. **(7p)** Sud težine G_1 koja pliva na površini neke tečnosti što je prikazano na slici, sadrži izvjesnu količinu iste tečnosti težine G_2. Odrediti težinu G tijela koji pliva u sudu iz uslova da odnos gaza H suda i visine h bude $H/h = n$.

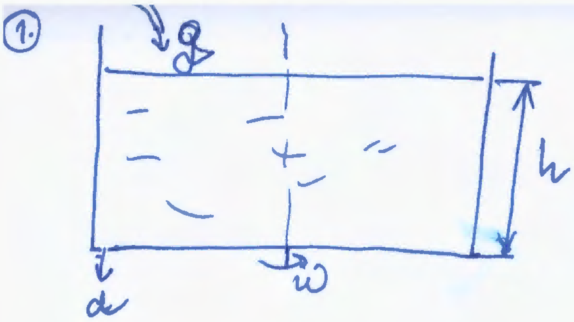


4. **(5p)** Za ravansko strujanje nestišljivog fluida, određeno potencijalom brzine

$\varphi(x, y) = x^3 + 6x^2y - 3xy^2 - 2y^3$., naći funkciju ψ (x, y), kao i komponente brzina za tačku A (1,1), i nagib strujnice.

Vidosava Vilotijević

1.



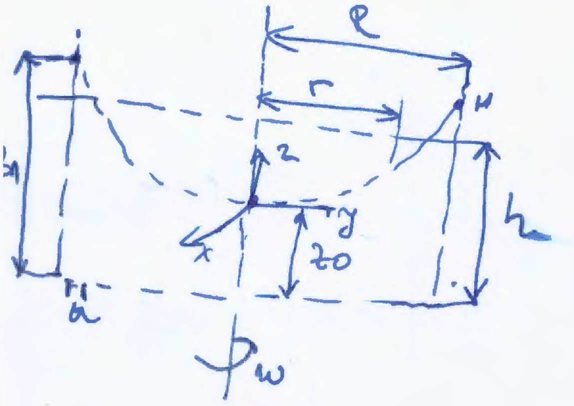
$$(x+x_{in})dx + (y+y_{in})dy + (z+z_{in})dz = \frac{dp}{\rho}$$

$$x = y = 0 \qquad z = -g$$

$$x_{in} = x\omega^2 \quad y_{in} = y\omega^2 \quad z_{in} = 0$$

$$x\omega^2 dx + y\omega^2 dy - g dz = \frac{dp}{\rho} \Big/ \int$$

$$\frac{x^2}{2}\omega^2 + \frac{y^2}{2}\omega^2 - gz = \frac{p}{\rho} + C.$$



$$\frac{r^2}{2}\omega^2 - gz = \frac{p}{\rho} + C.$$

$$x = y = z = 0 \quad p = p_b \quad C = -\frac{p_b}{\rho}$$

$$\boxed{\frac{r^2}{2}\omega^2 - gz = \frac{p - p_b}{\rho}}$$

Из једнакости запремина пре и после обртања:

$$R^2\pi \cdot h = R^2\pi z_0 + \frac{1}{2}R^2\pi(z_1 - z_0) \Rightarrow$$

$$\Rightarrow h = \frac{1}{2}(z_1 + z_0)$$

$$\Rightarrow z_1 = 2h + \frac{\omega^2 R^2}{4g}$$

Кад се суд обрће, проток кроз отвор површине a је

$$Q = \mu a\sqrt{2g z_1} \qquad q = Q$$

Брзина истицања течности.

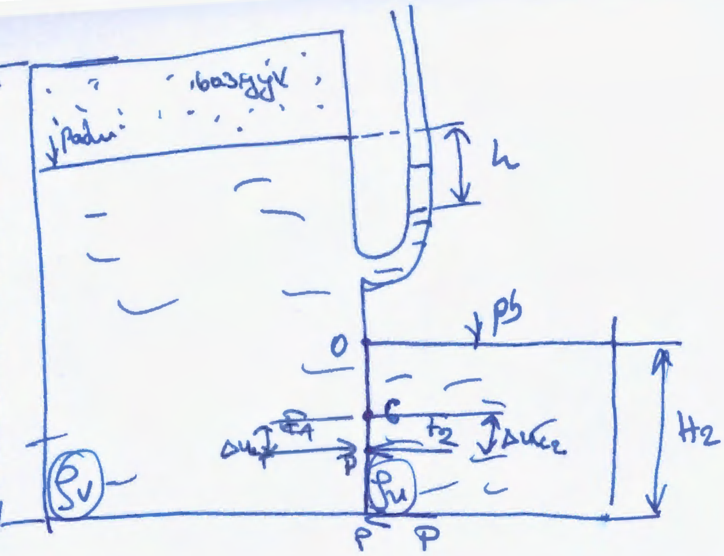
$$v = \varphi \cdot \sqrt{2g z_1}$$

$$Q = q = \mu \cdot a\sqrt{2gh + \frac{\omega^2 R^2}{2}}$$

$$\boxed{v = \varphi \cdot \sqrt{2gh + \frac{\omega^2 R^2}{2}}}$$

$H_1 = 5m$; $H_2 = 2m$; $h = 2m$; $\rho_v = 1000$

$\rho_u = 800\ kg/m^3$



$F_2 = \rho \cdot g \cdot z_{c2} \cdot A$

$= \rho_u \cdot g \cdot \frac{H_2}{2} \cdot H_2 \cdot 1 = 15{,}69\ kN$

$u_{c2} = z_{c2}$.

$\Delta z_{c2} = \frac{I_{yc2}}{u_{c2} \cdot A_2} = \frac{\frac{H_2^3 \cdot 1}{12}}{\frac{H_2}{2} \cdot H_2 \cdot b} = \frac{H_2}{6} = 0{,}333\ m$.

$F_1 = \rho_v \cdot g \cdot z_{c1} \cdot A$ $\qquad z_{c1} = H_1 - \frac{H_2}{2} - h$

$F_1 = \rho_v \cdot g \cdot (H_1 - \frac{H_2}{2} - h) \cdot H_2 \cdot 1 = 39{,}24\ kN$

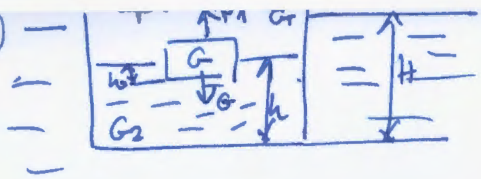
$u_{c1} = z_{c1}$ $\qquad \Delta u_{c1} = \frac{\frac{H_2^3 \cdot 1}{12}}{u_{c1} \cdot H_2 \cdot 1} = \underline{0{,}166\ m}$

$\Sigma M_0 = 0$

$F_1 \cdot (\frac{H_2}{2} + \Delta u_{c1}) - F_2 (\frac{H_2}{2} + \Delta u_{c2}) - P \cdot H_2 = 0$

$P = \frac{F_1 \cdot (\frac{H_2}{2} + \Delta u_{c1}) - F_2 (\frac{H_2}{2} + \Delta u_{c2})}{H_2} = \underline{\underline{12{,}415\ kN}}$

3



$P_1 = G \Rightarrow \underline{G = \rho \cdot g\, A_0 \cdot h_0}$

$P_2 = G_1 + G_2 + G$

$G_2 = \rho \cdot g \cdot A_1 \cdot h - \rho \cdot g\, A_0 \cdot h_0$

$G_2 = \rho \cdot g \cdot A_1 \cdot h - G$

$$G_1 + G_2 + G = \rho \cdot g \cdot A_1 \cdot H \Rightarrow G_1 + \rho \cdot g \cdot A_1 \cdot h - \cancel{G} + \cancel{G} = \rho \cdot g \cdot A_1 \cdot H$$

$$G_1 = \rho \cdot g \cdot A_1 \cdot H - \rho \cdot g \cdot A_1 \cdot h = \rho \cdot g \cdot A_1 (H - h)$$

$$\Rightarrow A_1 = \frac{G_1}{\rho g (H - h)}$$

$$G_2 = \rho \cdot g \cdot A_1 \cdot h - G \Rightarrow G_2 + G = \cancel{\rho g} \cdot h \cdot \frac{G_1}{\cancel{\rho g}(H-h)}$$

$$G_2 + G = \frac{G_1 \cdot h}{H - h} \Rightarrow G = \frac{G_1}{\frac{H-h}{h}} - G_2$$

$$\boxed{G = \frac{G_1}{h-1} - G_2}$$

4

$$\varphi(x,y) = x^3 + 6x^2y - 3xy^2 - 2y^3$$

$$v_x = \frac{\partial \varphi}{dx} = 3x^2 + 12xy - 3y^2$$

$$v_y = \frac{\partial \varphi}{\partial y} = 6x^2 - 6xy - 6y^2$$

$$v_x = \frac{\partial \psi}{\partial y} \Rightarrow \partial \psi = v_x dy \Rightarrow d\psi = (3x^2 + 12xy - 3y^2)dy \; // \int$$

$$\psi = 3x^2y + 6xy^2 - y^3 + f(x)$$

$$\frac{\partial \psi}{\partial x} = \cancel{6xy} + \cancel{6y^2} + f'(x) = -v_y = -6x^2 + \cancel{6xy} + \cancel{6y^2}$$

$$f'(x) = -6x^2 \Rightarrow \boxed{f(x) = -2x^3 + c}$$

$$\boxed{\psi = 3yx^2 + 6xy^2 - y^3 - 2x^3 + c}$$

$v_{xA} = 3 + 12 - 3 = 12$ m/s

$v_{yA} = 6 - 6 - 6 = -6$ m/s

нагиб струјнице:

$$\left(\frac{dy}{dx}\right)_A = \left(\frac{v_y}{v_x}\right)_A = \left(\frac{-6}{12}\right) = -\frac{1}{2}.$$