Парцијални изводи

$$f(x,y) = \frac{x^2+y^2}{xy^2}$$

$$\frac{\partial f}{\partial x} = \frac{1}{y^2} \cdot \frac{2x \cdot x - (x^2+y^2)\cdot 1}{x^2}$$

$$\boxed{\frac{\partial f}{\partial x} = \frac{1}{y^2} \cdot \frac{x^2-y^2}{x^2}}$$

$$\frac{\partial f}{\partial y} = \frac{1}{x} \cdot \frac{2y y^2 - (x^2+y^2)\cdot 2y}{y^4}$$

$$\boxed{\frac{\partial f}{\partial y} = \frac{1}{x} \cdot \frac{-2x^2 y}{y^4} = -\frac{2x}{y^3}}$$

$$f(x,y,z) = \left(\frac{x}{y}\right)^z$$

$$\frac{\partial f}{\partial x} = \frac{x^z}{y^z} = \frac{1}{y^z} \cdot z \cdot x^{z-1}$$

$$\frac{\partial f}{\partial y} = x^z \cdot (-z) \cdot y^{-z-1} = -z \cdot x^z \cdot y^{z+1}$$

$$\frac{\partial f}{\partial z} = \left(\frac{x}{y}\right)^z \cdot \ln\left(\frac{x}{y}\right)$$

Ако је $f(x,y) = \operatorname{arctg} \frac{y}{x}$ наћи $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y^2}$ (парцијални изводи другог реда)

$$\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{y^2}{x^2}\right)} \cdot y \cdot \left(-\frac{1}{x^2}\right) = \frac{\frac{-y}{\cancel{x^2}}}{\frac{x^2+y^2}{\cancel{x^2}}} = \frac{-y}{x^2+y^2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x} \cdot 1 = \frac{x}{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial f}{\partial x}\left(-\frac{y}{x^2+y^2}\right) =$$

$$= -y(-1)(x^2+y^2)^{-2} \cdot 2x = 2xy/(x^2+y^2)^2$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial f}{\partial y}\left(\frac{x}{x^2+y^2}\right) = x \cdot (-1)(x^2+y^2)^{-2} \cdot 2y =$$

$$\boxed{= \frac{-2xy}{(x^2+y^2)^2}}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) = \frac{1 \cdot (x^2+y^2) - x \cdot 2x}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

Тотални диференцијал I реда

$$f = f(x,y)$$

$$df = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy$$

Scanned by TapScanner

$$f = f(x,y,z)$$

$$df = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy + \frac{\partial f}{\partial z} \cdot dz$$

② Наћи диференцијал I реда функције

$$f(x,y) = \frac{xy}{x-y}$$

$$\frac{\partial f}{\partial x} = \frac{y \cdot (x-y) - xy \cdot 1}{(x-y)^2} = \frac{-y^2}{(x-y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{x \cdot x-y-y\cdot(-1)}{(x-y)^2} = \frac{x^2}{(x-y)^2}$$

$$df = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy$$

$$df = \frac{-y^2}{(x-y)^2} \cdot dx + \frac{x^2}{(x-y)^2} \cdot dy$$

③ Наћи диференцијал I реда функције

$$f(x,y,z) = \frac{z}{x^2+y^2}$$

$$\frac{\partial f}{\partial x} = z(-1) \cdot (x^2+y^2)^{-2} \cdot 2x = -\frac{2xz}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = z \cdot (-1) \cdot (x^2+y^2)^{-2} \cdot 2y = \frac{-2y \cdot z}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial z} = \frac{1}{x^2+y^2}$$

$$df = \frac{-2xz}{(x^2+y^2)^2} \cdot dx + \frac{-2yz}{(x^2+y^2)^2} \cdot dy + \frac{1}{x^2+y^2} dz$$

Наћи потпуни диференцијал II реда за функцију.

$$f(x,y) = x^3+y^3-3x^2y+3xy^2$$

$$d^2f = \frac{\partial^2 f}{\partial x^2} \cdot dx^2 + 2\frac{\partial^2 f}{\partial x \cdot \partial y} \cdot dx \cdot dy + \frac{\partial^2 f}{\partial y^2} \cdot dy^2$$

$$\frac{\partial f}{\partial x} = 3x^2-6xy+3y^2$$

$$\frac{\partial^2 f}{\partial x^2} = 6x-6y$$

$$\frac{\partial f}{\partial y} = 3y^2-3x^2+6xy$$

$$\frac{\partial^2 f}{\partial y^2} = 6y+6x$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = -6x+6y$$

$$\boxed{d^2f = 6(x-y) \cdot dx^2 + 2 \cdot 6(y-x) \cdot dx \cdot dy + 6(y+x)\, dy^2}$$

Наћи $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ ако је $f(u,v) = u^2 \ln v$

$$u = \frac{x}{y} \qquad v = 3x-2y$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial f}{\partial x} = 2u \cdot \ln v \, \frac{1}{y} + u^2 \cdot \frac{1}{v} \cdot 3$$

$$\frac{\partial f}{\partial x} = \frac{2x}{y} \cdot \ln(3x-2y) \cdot \frac{1}{y} + \frac{x^2}{y^2} \cdot \frac{1}{3x-2y} \cdot 3$$

$$\boxed{\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial y}}$$

$$\frac{\partial f}{\partial y} = 2u \cdot \ln v \left(-\frac{x}{y^2}\right) + u^2 \cdot \frac{1}{v}(-2)$$

$$\frac{\partial f}{\partial y} = \frac{2x}{y}\left(-\frac{x}{y^2}\right) \cdot \ln(3x-2y) - 2 \cdot \frac{x^2}{y^2} \cdot \frac{1}{3x-2y}$$

● Нађи d^2f за $f = f(u,v)$ где је $u = x^2 \cdot y + y^3$

$v = x^2 y - y^2$

$$d^2f = \frac{\partial^2 f}{\partial x^2} \cdot dx^2 + 2\frac{\partial^2 f}{\partial x \partial y} \cdot dx\,dy + \frac{\partial^2 f}{\partial y^2} \cdot dy^2$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial x} = 2x^2 y$$

$$\frac{\partial u}{\partial y} = x^2 + 3y^2$$

$$\frac{dV}{dx} = 2xy$$

$$\frac{dV}{dy} = x^2 - 2y$$

$$\frac{df}{dx} = \frac{df}{du} \cdot 2xy + \frac{df}{dx} \cdot 2xy$$

$$\frac{df}{dx} = 2xy \cdot \frac{df}{du} + 2xy \cdot \frac{df}{dx}$$

извод производа

$$\frac{d^2f}{dx^2} = \frac{d}{dx}\left(\frac{df}{dx}\right) = \frac{d}{dx}\left(2xy \cdot \frac{df}{du} + 2xy \cdot \frac{df}{dv}\right)$$

$$= 2y \cdot \frac{df}{du} + 2xy\left(\frac{d^2f}{du^2} \cdot \frac{du}{dx} + \frac{d^2f}{dv\,du} \cdot \frac{dV}{dx}\right) +$$

$$+ 2y \cdot \frac{df}{dx} + 2xy\left(\frac{df}{du \cdot dv} \cdot \frac{du}{dx} + \frac{d^2f}{dx^2} \cdot \frac{d}{dx}\right)$$

$\Downarrow$ уврстимо

Наћи y' имплицитно задате функције

$$F(x,y) = x^2 + xy + y^2 = 3$$

$$\frac{dF}{dx} \cdot 1 + \frac{dF}{dy} \cdot \frac{dy}{dx} = 0$$

$$y'_x = \frac{F'x}{F'y}$$

$$\left.\begin{aligned} F'(x) &= 2x + y \\ F'(y) &= x + 2y \end{aligned}\right\} \; y'_x = -\frac{2x+y}{x+2y}$$