

$$1^{\circ} \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$$2^{\circ} \lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$$

$$3^{\circ} \lim_{n \rightarrow \infty} \sqrt[n]{a} = 1, a > 0$$

$$4^{\circ} \lim_{n \rightarrow \infty} \frac{n^k}{a^n} = 0, a > 0$$

$$5^{\circ} \lim_{n \rightarrow \infty} \frac{n^a}{n!} = 0, a \in \mathbb{R}$$

$$6^{\circ} \lim_{n \rightarrow \infty} \frac{an}{n!} = 0$$

$$7^{\circ} \lim_{n \rightarrow \infty} q^n = \begin{cases} 0 & |q| < 1 \\ 1 & q = 1 \\ +\infty & q > 1 \end{cases}$$

$$\frac{5}{6} \rightarrow 0$$

$$1) \lim_{n \rightarrow \infty} \frac{5n^2 + 2n}{n^3 + 1} = \lim_{n \rightarrow \infty} \frac{n^2(5 + \frac{2}{n})}{n^3(1 + \frac{1}{n^3})} =$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \cdot \frac{5 + \frac{2}{n}}{1 + \frac{1}{n^3}} = 0$$

$$2) \lim_{x \rightarrow \infty} \frac{(2n+1)(n+1) \cdot n}{n^3 + 3n^2 + 3n + 1} = \lim_{x \rightarrow \infty} \frac{(2n+1)(n^2+n)}{n^3 + 3n^2 + 3n + 1} =$$

$$= \lim_{x \rightarrow \infty} \frac{2n^3 + 3n^2 + n}{n^3 + 3n^2 + 1} = \lim_{n \rightarrow \infty} \frac{n^3(2 + \frac{3}{n} + \frac{1}{n^2})}{n^3(1 + \frac{3}{n} + \frac{1}{n^2})} = 2$$

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$$\textcircled{3} \lim_{n \rightarrow \infty} \frac{1+2+\dots+n}{n^2} = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2}}{n^2} =$$

$$= \lim_{n \rightarrow \infty} \frac{n^2+n}{2n^2} = \lim_{n \rightarrow \infty} \frac{n^2(1+\frac{1}{n})}{2n^2} = \frac{1}{2}$$

$$\textcircled{4} \lim_{n \rightarrow \infty} \frac{1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1)}{n^3} = \begin{cases} n(n+1) = n^2+n \\ n=1 & 1 \cdot 2 = 1^2+1 \\ n=2 & 2 \cdot 3 = 2^2+2 \\ n=3 & 3 \cdot 4 = 3^2+3 \end{cases}$$

$$\lim_{n \rightarrow \infty} \frac{1^2+1+2^2+2+3^2+3+\dots+n^2+n}{n^3}$$

$$\lim_{n \rightarrow \infty} \frac{1+2+\dots+n+1^2+2^2+\dots+n^2}{n^3} =$$

$$= \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6}}{n^3}$$

$$\lim_{n \rightarrow \infty} \frac{3n^2+3n+2n^3+3n^2+n}{6n^3} = \lim_{n \rightarrow \infty} \frac{2n^3+6n^2+4n}{6n^3} =$$

$$= \lim_{n \rightarrow \infty} \frac{n^3(2+\frac{6}{n}+\frac{4}{n^2})}{6n^3} = \lim_{n \rightarrow \infty} \frac{1}{3}$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$⑤ \lim_{n \rightarrow \infty} (n-2 - \sqrt{n^2 - n + 1}) = \lim_{n \rightarrow \infty} (n-2 - \sqrt{n^2 - n + 1}) \cdot \frac{n-2 + \sqrt{n^2 - n + 1}}{n-2 + \sqrt{n^2 - n + 1}}$$

$$= \lim_{n \rightarrow \infty} \frac{(n-2)^2 - (n^2 - n + 1)}{n-2 + \sqrt{n^2 - n + 1}} = \lim_{n \rightarrow \infty} \frac{-3n + 3}{n-2 + \sqrt{n^2(1 - \frac{1}{n} + \frac{1}{n^2})}}$$

$$= \lim_{n \rightarrow \infty} \frac{3n \left(\frac{1}{n} - 1 \right)}{n \left(1 - \frac{2}{n} + \sqrt{1 - \frac{1}{n} + \frac{1}{n^2}} \right)} = -\frac{3}{2}$$

$$⑥ \lim_{n \rightarrow \infty} \sqrt[3]{n^2} \cdot (\sqrt[3]{n-1} - \sqrt[3]{n+1}) = \text{~~scribble~~}$$

$$= \lim_{n \rightarrow \infty} \sqrt[3]{n^2} \cdot (\sqrt[3]{n-1} - \sqrt[3]{n+1}) \cdot \frac{\sqrt[3]{(n-1)^2} + \sqrt[3]{n^2-1} + \sqrt[3]{(n+1)^2}}{\sqrt[3]{(n-1)^2} + \sqrt[3]{n^2-1} + \sqrt[3]{(n+1)^2}} =$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2} (n-1 - (n+1))}{\sqrt[3]{n^2} (\sqrt[3]{1 - \frac{2}{n} + \frac{1}{n^2}} + \sqrt[3]{1 - \frac{1}{n}} + \sqrt[3]{1 + \frac{2}{n} + \frac{1}{n^2}})} = -\frac{2}{3}$$

$$⑦ \lim_{n \rightarrow \infty} \frac{(-2)^n + 5^n}{(-2)^{n+1} + 5^{n+1}} = \lim_{n \rightarrow \infty} \frac{5^n \left(\left(-\frac{2}{5}\right)^n + 1 \right)}{5^{n+1} \left(\left(-\frac{2}{5}\right)^{n+1} + 1 \right)} =$$

$$= \lim_{n \rightarrow \infty} \frac{1}{5} \cdot \frac{\left(-\frac{2}{5}\right)^n + 1}{\left(-\frac{2}{5}\right)^{n+1} + 1} = \frac{1}{5}$$

$$⑧ \lim_{n \rightarrow \infty} \frac{1+2+2^2+\dots+2^n}{1+3+3^2+\dots+3^n} = \lim_{n \rightarrow \infty} \frac{1 \cdot \frac{2^{n+1} - 1}{2 - 1}}{1 \cdot \frac{3^{n+1} - 1}{3 - 1}} =$$

$$2 \lim_{n \rightarrow \infty} \frac{2^{n+1} - 1}{3^{n+1} - 1} = 2 \cdot \lim_{n \rightarrow \infty} \frac{2^{n+1} \left(1 - \frac{1}{2^{n+1}} \right)}{3^{n+1} \left(1 - \frac{1}{3^{n+1}} \right)} =$$

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$$2. \lim_{n \rightarrow \infty} \left(\frac{2}{3}\right)^{n+1} \cdot \lim_{n \rightarrow \infty} \frac{1 - \frac{1}{2^{n+1}}}{1 - \frac{1}{2^{n+1}}} = 0$$

$$9. \lim_{n \rightarrow \infty} \sqrt{2} \cdot \sqrt[4]{2} \cdot \sqrt[8]{2} \dots \sqrt[2^n]{2} =$$

$$\lim_{n \rightarrow \infty} 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2^2}} \cdot 2^{\frac{1}{2^3}} \dots 2^{\frac{1}{2^n}} =$$

$$\lim_{n \rightarrow \infty} 2^{\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^n}} =$$

$$\lim_{n \rightarrow \infty} 2^{\frac{1}{2} \cdot \frac{1 - (\frac{1}{2})^n}{1 - \frac{1}{2}}} = \lim_{n \rightarrow \infty} 2^{1 - (\frac{1}{2})^n} = 2$$

10. Определите сумму бесконечной числовой последовательности
 определите член $a_n = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)}$

$$a_n = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)}$$

$$a_1 = \frac{1}{2}$$

$$a_2 = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3}$$

$$a_3 = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4}$$

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$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$\frac{1}{n(n+1)} = \frac{An + A + Bn}{n(n+1)}$$

$$An + A + Bn = 1 \quad \# \# \# \quad \#$$

$$\left. \begin{aligned} (A+B)n + A &= 1 \Rightarrow A = 1 \\ A+B &= 0 \end{aligned} \right\} \Rightarrow \begin{aligned} A &= 1 \\ B &= -1 \end{aligned}$$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$n=1 \quad \frac{1}{1 \cdot 2} = \frac{1}{1} - \frac{1}{2}$$

$$n=2 \quad \frac{1}{2 \cdot 3} = \frac{1}{2} - \frac{1}{3}$$

$$n=3 \quad \frac{1}{3 \cdot 4} = \frac{1}{3} - \frac{1}{4}$$

$$n=4 \quad \frac{1}{4 \cdot 5} = \frac{1}{4} - \frac{1}{5}$$

$$a_n = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{n} - \frac{1}{n+1}$$

$$a_n = 1 - \frac{1}{n+1}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1} \right) = 1$$

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① Примером теореме о мањој мањој доказивању конвергенцију и наћи граничну вредност низа, који је облика x_n .

$$a) x_n = \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}}$$

$$\underline{R} \quad a_n \leq x_n \leq b_n, \quad \forall n \in \mathbb{N}$$

$$\left. \begin{array}{l} \lim_{n \rightarrow \infty} a_n = l \\ \lim_{n \rightarrow \infty} b_n = l \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} x_n = l$$

$$x_n = \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}} \leq$$

$$\leq \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+1}} + \dots + \frac{1}{\sqrt{n^2+1}} =$$

$$\frac{n \cdot 1}{\sqrt{n^2+1}}$$

$$x_n \geq \frac{1}{\sqrt{n^2+n}} + \frac{1}{\sqrt{n^2+n}} + \dots + \frac{1}{\sqrt{n^2+n}} = n \cdot \frac{1}{\sqrt{n^2+n}}$$

Ako je $a_n = \frac{n}{\sqrt{n^2+n}}$ $b_n = \frac{n}{\sqrt{n^2+1}}$

Ugodi je

1° $a_n \leq x_n \leq b_n, \forall n \in \mathbb{N}$

2° $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2+n}} = \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2(1+\frac{1}{n})}} = \lim_{n \rightarrow \infty} \frac{n}{n \cdot \sqrt{1+\frac{1}{n}}} = 1$

3° $\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n \rightarrow \infty} \frac{n}{n \cdot \sqrt{1+\frac{1}{n^2}}} = 1$

Uz 1°, 2°, 3° uz ostale teoreme o uklopljenosti
slijedi da niz (x_n) konvergira i da je

$\lim_{n \rightarrow \infty} x_n = 1$

5) $x_n = \sum_{k=1}^n \frac{5k}{3n^2-2k}$

R $x_n = \frac{5n}{3n^2-2} + \frac{5n}{3n^2-4} + \frac{5n}{3n^2-6} + \dots + \frac{5n}{3n^2-2n}$

$x_n \leq \frac{5n}{3n^2-2n} + \frac{5n}{3n^2-2n} + \dots + \frac{5n}{3n^2-2n} =$

$n \cdot \frac{5n}{3n^2-2n} = \frac{5n}{3n^2-2n}$

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$$x_n \geq \frac{5n}{3n^2-2} + \frac{5n}{3n^2-2} + \dots + \frac{5n}{3n^2-2} = \frac{n \cdot 5n}{3n^2-2} = \frac{5n^2}{3n^2-2n}$$

Heka je $a_n = \frac{5n^2}{3n^2-2n}$ u $b_n = \frac{5n^2}{3n^2-2n}$

Stoga je

1° $a_n \leq x_n \leq b_n, \forall n \in \mathbb{N}$

2° $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5n^2}{3n^2-2} = \frac{5}{3}$

3° $\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{5n^2}{3n^2-2n} = \frac{5}{3}$

Uz 1°, 2°, 3° slijedi da niz (x_n) konvergira u $\lim_{n \rightarrow \infty} x_n = \frac{5}{3}$

Далее

$$x_n = \sum_{k=1}^n \frac{5k}{3n^2-2k}$$

$$x_n \leq \sum_{k=1}^n \frac{5n}{3n^2-2n} = \frac{n \cdot 5n}{3n^2-2n}$$

$$x_n \geq \sum_{k=1}^n \frac{5k}{3n^2-2} = \frac{n \cdot 5n}{3n^2-2}$$

$$a_n = \frac{n+(-1)^1}{\sqrt[5]{n^{10}+1}} + \frac{n+(-1)^2}{\sqrt[5]{n^{10}+2}} + \dots + \frac{n+(-1)^n}{\sqrt[5]{n^{10}+n}}$$

$$a_n \leq \frac{n+1}{\sqrt[5]{n^{10}+1}} + \frac{n+1}{\sqrt[5]{n^{10}+1}} + \dots + \frac{n+1}{\sqrt[5]{n^{10}+1}} = n \cdot \frac{n+1}{\sqrt[5]{n^{10}+1}}$$

$$a_n \geq \frac{n-1}{\sqrt[5]{n^{10}+n}} + \frac{n-1}{\sqrt[5]{n^{10}+n}} + \dots + \frac{n-1}{\sqrt[5]{n^{10}+n}} = n \cdot \frac{n-1}{\sqrt[5]{n^{10}+n}}$$

$$x_n = n \cdot \frac{n-1}{\sqrt[5]{n^{10}+n}}$$

$$y_n = n \cdot \frac{n+1}{\sqrt[5]{n^{10}+1}}$$

Itaga je

$$1^\circ x_n \leq a_n \leq y_n$$

$$2^\circ \lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} \frac{n^2 - n}{\sqrt[5]{n^{10} \left(1 + \frac{1}{n^9}\right)}} = \lim_{n \rightarrow \infty} \frac{n^2 \left(1 - \frac{1}{n}\right)}{n^2 \cdot \sqrt[5]{1 + \frac{1}{n^9}}} = \underline{1}$$

$$3^\circ \lim_{n \rightarrow \infty} y_n = \lim_{n \rightarrow \infty} \frac{n^2 + n}{\sqrt[5]{n^{10} \left(1 + \frac{1}{n^9}\right)}} = \lim_{n \rightarrow \infty} \frac{n^2 \left(1 + \frac{1}{n}\right)}{n^2 \cdot \sqrt[5]{1 + \frac{1}{n^9}}} = \underline{1}$$

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Из $1^0, 2^0, 3^0$ следи да је $\lim a_n = \underline{1}$

12) Приметити ширине о ефикасности и одређити
 низу доказати конвергенцију низа.

$$a) a_n = \frac{1}{3+1} + \frac{1}{3^2+1} + \dots + \frac{1}{3^n+1}$$

$$\begin{aligned} \text{Р} // a_{n+1} - a_n &= \frac{1}{3+1} + \frac{1}{3^2+1} + \dots + \frac{1}{3^n+1} + \frac{1}{3^{n+1}+1} \\ &\quad - \frac{1}{3+1} - \frac{1}{3^2+1} - \dots - \frac{1}{3^n+1} \end{aligned}$$

$$a_{n+1} - a_n = \frac{1}{3^{n+1}+1} > 0, \forall n \in \mathbb{N}$$

$$a_{n+1} - a_n > 0, \forall n \in \mathbb{N}$$

$a_{n+1} > a_n, \forall n \in \mathbb{N} \Rightarrow$ низ (a_n) строго монотонно
 расте.

Како је низ строго монотонно расте, он
 је $a_1 \leq a_n, \forall n \in \mathbb{N}$

$$\frac{1}{4} \leq a_n \quad \forall n \in \mathbb{N}$$

$$a_n = \frac{1}{3+1} + \frac{1}{3^2+1} + \dots + \frac{1}{3^n+1} \leq \frac{1}{3} + \frac{1}{3^2} + \dots + \frac{1}{3^n} =$$

$$= \frac{1}{3} \cdot \frac{1 - \left(\frac{1}{3}\right)^n}{1 - \frac{1}{3}} = \frac{1}{3} \cdot \frac{3}{2} \cdot \left(1 - \left(\frac{1}{3}\right)^n\right)$$

$$a_n \leq \frac{1}{2} \left(1 - \left(\frac{1}{3}\right)^n\right) < \frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$\frac{1}{4} \leq a_n \leq \frac{1}{2} \text{ низ } (a_n) \text{ је обратан}$$

1°) (a_n) је монотон низ } низ (a_n) конвергентан
 2°) (a_n) је обратан }

$$b) a_n = \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{4}\right) \cdot \dots \cdot \left(1 - \frac{1}{2^{n+1}}\right)$$

$$\frac{a_{n+1}}{a_n} = \frac{\left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{2^{n+1}}\right) \cdot \left(1 - \frac{1}{2^{n+2}}\right)}{\left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{2^{n+1}}\right)}$$

$$\frac{a_{n+1}}{a_n} = 1 - \frac{1}{2^{n+2}}, \quad \forall n \in \mathbb{N}$$

$$\left. \begin{array}{l} \frac{a_{n+1}}{a_n} < 1, \quad \forall n \in \mathbb{N} \\ a_n > 0, \quad \forall n \in \mathbb{N} \end{array} \right\} \Rightarrow a_{n+1} < a_n, \quad \forall n \in \mathbb{N}$$

\Rightarrow низ (a_n) је строго монотонно
 опадајући

Како је низ (a_n) строго опадајући он је

$$a_n \leq a_1, \quad \forall n \in \mathbb{N}$$

$$a_1 = \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{4}\right) = \frac{3}{8}$$

$$a_n \leq \frac{3}{8}, \quad \forall n \in \mathbb{N}$$

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$0 < a_n \leq \frac{3}{8}$, $\forall n \in \mathbb{N} \Rightarrow (a_n)$ је опадajuћи

1° (a_n) еламентаран } $\Rightarrow (a_n)$ је конвергентан
 2° (a_n) опадajuћи

$$b) x_n = \frac{\ln(1+1)}{1 \cdot 2} + \frac{\ln(1+\frac{1}{2})}{2 \cdot 3} + \dots + \frac{\ln(1+\frac{1}{n})}{n(n+1)}$$

$$x_n \leq \frac{\ln 2}{1 \cdot 2} + \frac{\ln 2}{2 \cdot 3} + \dots + \frac{\ln 2}{n(n+1)} =$$

$$= \ln 2 \cdot \left(\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)} \right) =$$

$$= \ln 2 \left(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{n} - \frac{1}{n+1} \right) =$$

$$= \ln 2 \left(1 - \frac{1}{n+1} \right)$$

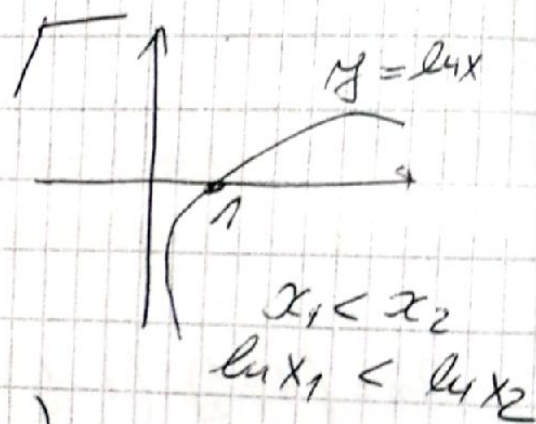
$$x_n \leq \ln 2 \cdot \left(1 - \frac{1}{n+1} \right)$$

$$x_n \leq \ln 2 \cdot 1, \forall n \in \mathbb{N}$$

$0 < x_n \leq \ln 2$, $\forall n \in \mathbb{N} \Rightarrow (x_n)$ - опадajuћи

$$x_{n+1} - x_n = \frac{\ln(1+\frac{1}{n+1})}{(n+1)(n+2)} > 0, \forall n \in \mathbb{N}$$

$$x_{n+1} > x_n, \forall n \in \mathbb{N}$$



Низ (x_n) је строго екстремно растући
 1° (x_n) екстремно } $\Rightarrow (x_n)$ је конвергентан низ
 2° (x_n) -обратан }

13) Наћи граничну вредност низа који је
 општи члан

$$x_n = \sqrt[n]{4^n + 3^n - \sin(n!)}$$

$$\underline{R} \quad x_n = \sqrt[n]{4^n \cdot \left(1 + \left(\frac{3}{4}\right)^n - \frac{\sin(n!)}{4^n}\right)} = 4 \cdot \sqrt[n]{1 + \left(\frac{3}{4}\right)^n - \frac{\sin(n!)}{4^n}}$$

$$x_n \leq 4 \cdot \sqrt[n]{1 + \frac{3}{4} + \frac{1}{4}}$$

$$x_n \leq 4 \cdot \sqrt[n]{2}$$

$$x_n \geq 4 \cdot \sqrt[n]{1 + 0 - \frac{1}{4}} = 4 \cdot \sqrt[n]{\frac{3}{4}}$$

$$1^\circ \quad 4 \cdot \sqrt[n]{\frac{3}{4}} < x_n \leq 4 \cdot \sqrt[n]{2}$$

$$2^\circ \quad \lim_{n \rightarrow \infty} 4 \sqrt[n]{2} = 4 \cdot 1 = 4$$

$$3^\circ \quad \lim_{n \rightarrow \infty} 4 \sqrt[n]{\frac{3}{4}} = 4 \cdot 1 = 4$$

Из 1°, 2°, 3° следи да је низ конвергентан

$$\text{и} \quad \lim_{n \rightarrow \infty} x_n = 4$$

$$\textcircled{1} \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n = \lim_{n \rightarrow \infty} \left(\left(\frac{n+1}{n} \right)^{-1} \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^{-n} = e^{-1} = \frac{1}{e}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = e$$

$$\textcircled{2} \lim_{n \rightarrow \infty} \left(\frac{n+1}{n-1} \right)^n \stackrel{\infty}{=} \lim_{n \rightarrow \infty} \left(1 + \frac{n+1}{n-1} - 1 \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n-1} \right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n-1} \right)^{\frac{n-1}{2} \cdot \frac{2}{n-1} n}$$

$$= \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n-1} \right)^{\frac{n-1}{2}} \stackrel{\frac{2n}{n-1}}{=} \lim_{n \rightarrow \infty} \frac{2n}{2n-1} = e$$

$$\stackrel{(*)}{=} \lim_{n \rightarrow \infty} \frac{2n}{n(2 - \frac{1}{n})} = e$$

$$(*) : \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n-1} \right)^{\frac{n-1}{2}} \quad \left[\begin{array}{l} \frac{n-1}{2} = nu \\ n \rightarrow \infty \Rightarrow nu \rightarrow \infty \end{array} \right]$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{nu} \right)^{nu} = e$$

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