$$\alpha = \frac{\xi+\eta}{2},\ \beta = \frac{\xi-\eta}{2},\ \ (\alpha = \varphi_1(x,y),\ \beta = \varphi_2(x,y))$$

tako da je $\xi = \alpha + i\beta$, $\eta = \alpha - i\beta$. Tada je

$$\frac{\partial^2 z}{\partial\xi\partial\eta} = \frac{\partial}{\partial\eta}\left(\frac{\partial z}{\partial\xi}\right) = \frac{\partial}{\partial\eta}\left(\frac{\partial z}{\partial\alpha}\frac{\partial\alpha}{\partial\xi} + \frac{\partial z}{\partial\beta}\frac{\partial\beta}{\partial\xi}\right) = \frac{\partial}{\partial\eta}\left(\frac{\partial z}{\partial\alpha}\cdot\frac{1}{2} + \frac{\partial z}{\partial\beta}\cdot\frac{1}{2i}\right) =$$

$$\frac{1}{2}\left(\frac{\partial^2 z}{\partial\alpha^2}\cdot\frac{1}{2} + \frac{\partial^2 z}{\partial\alpha\partial\beta}\cdot\left(-\frac{1}{2i}\right)\right) + \frac{1}{2i}\left(\frac{\partial^2 z}{\partial\beta\partial\alpha}\cdot\frac{1}{2} + \frac{\partial^2 z}{\partial\beta^2}\cdot\left(-\frac{1}{2i}\right)\right) = \frac{1}{4}\left(\frac{\partial^2 z}{\partial\alpha^2} + \frac{\partial^2 z}{\partial\beta^2}\right) =$$

$$F_3\left(\xi,\eta,z,\frac{\partial z}{\partial\xi},\frac{\partial z}{\partial\eta}\right),$$

ili

$$\frac{\partial^2 z}{\partial\alpha^2} + \frac{\partial^2 z}{\partial\beta^2} = F_3\left(\alpha,\beta,z,\frac{\partial z}{\partial\alpha},\frac{\partial z}{\partial\beta}\right).$$

Ovo je kanonski oblik jednačine eliptičkog tipa.

Primjer 4. Odrediti tip PDJ $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x\partial y} + 5\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} = 0$ i svesti je na kanonski oblik.

Kako je A=1, B=2, C=5 i $\Delta = B^2 - AC = -1 < 0$, to je data jednačina eliptičkog tipa. Iz jednačine karakteristika $(dy)^2 - 4dxdy + 5(dx)^2 = 0$ dobijamo da je $(y-2x) \pm ix = C_{1/2}$. Uvedimo nove promjenljive $\xi = y - 2x$ i $\eta = x$. U novim promjenljivim data jednačina ima kanonski oblik $\frac{\partial^2 z}{\partial\xi^2} + \frac{\partial^2 z}{\partial\eta^2} = -\frac{\partial z}{\partial\eta}$.

Na kraju ovog odjeljka navedimo nekoliko zadataka za samostalni rad:

1. Riješiti jednačine: a) $\frac{\partial^2 z}{\partial x\partial y} = \frac{x}{y}$, b) $\frac{\partial^2 z}{\partial x\partial y} = 2\frac{\partial z}{\partial y}$, c) $\frac{\partial^2 z}{\partial x\partial y} + \frac{1}{x}\frac{\partial z}{\partial y} = 0$.
2. Odrediti tip PDJ i svesti ih na kanonski oblik:

   a) $y\frac{\partial^2 z}{\partial x^2} - x\frac{\partial^2 z}{\partial x\partial y} + 2\frac{\partial z}{\partial y} = 0$, b) $y^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + x^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$,

   c) $y^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + 2x^2\frac{\partial^2 z}{\partial y^2} + y\frac{\partial z}{\partial y} = 0$.