Primjena izvoda

$y = f(x)$

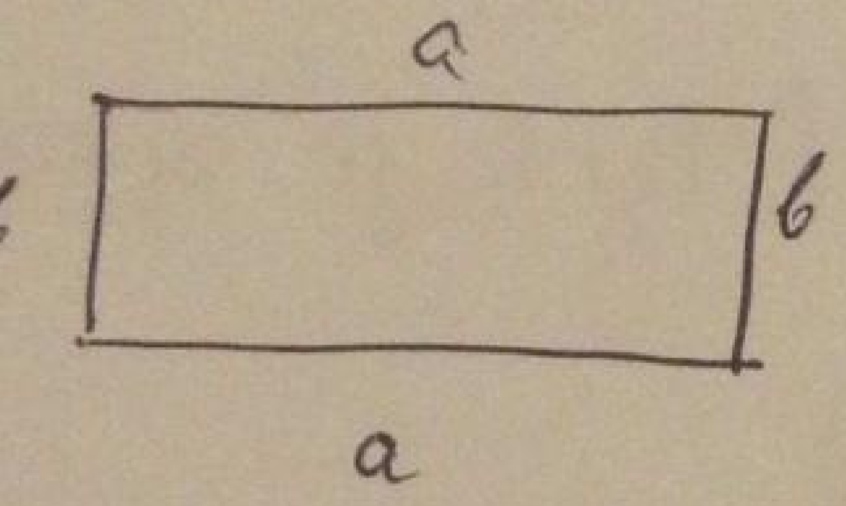
$f'(x)=0 \Rightarrow$ rješenje ove jednačine stacionarne tačke

$f''(x)>0 \Rightarrow x$ je t. lok. min.

$f''(x)<0 \Rightarrow x$ je t. lok. max

1) Od svih pravougaonika obima 20, odrediti onaj čija je površina maksimalna.

Rj:



$O = 2a+2b$

$2(a+b)=20$

$\underline{a+b=10}$

$P \to max;$ $P=a\cdot b$ (treba površinu izraziti kao funkciju jedne promjenljive)

$b=10-a \Rightarrow P=a\cdot(10-a) = 10a-a^2 = P(a) \Rightarrow P$ je fja od a.

$P'=0 \Leftrightarrow P'=10-2a=0$

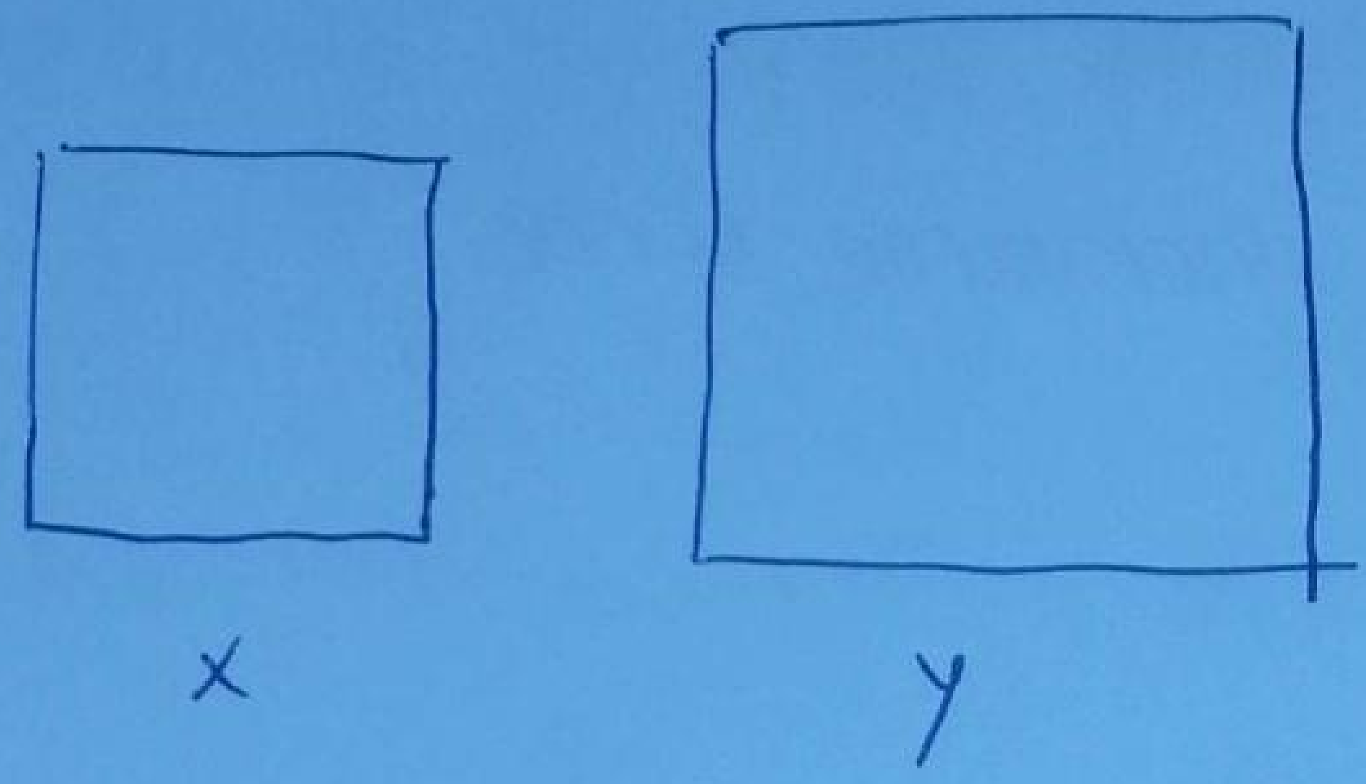
$2a=10$

$\underline{a=5}$

$P''=-2<0 \Rightarrow a=5$ je dužina stranice a $(b=10-5=5)$ za koju je P pravoug. maksimalna. $(P_{max}=25)$.

2) Odredi dužine stranica dva kvadrata tako da je njihov zbir 14, a zbir površina tih kvadrata minimalan.

Rj:



$x+y=14 \Rightarrow y=14-x$

$P_1+P_2 \rightarrow min.$

$P_1=x^2 \qquad P_2=y^2$

$\underbrace{P_1+P_2}_{P} = x^2+(14-x)^2 \rightarrow min$

$P' = 2x+2(14-x)\cdot(-1)$

$P' = 2x-2(14-x)$

$P' = 2x-28+2x$

$\underline{P' = 4x-28} \qquad P'=0$

$4x=28$

$\boxed{x=7}$

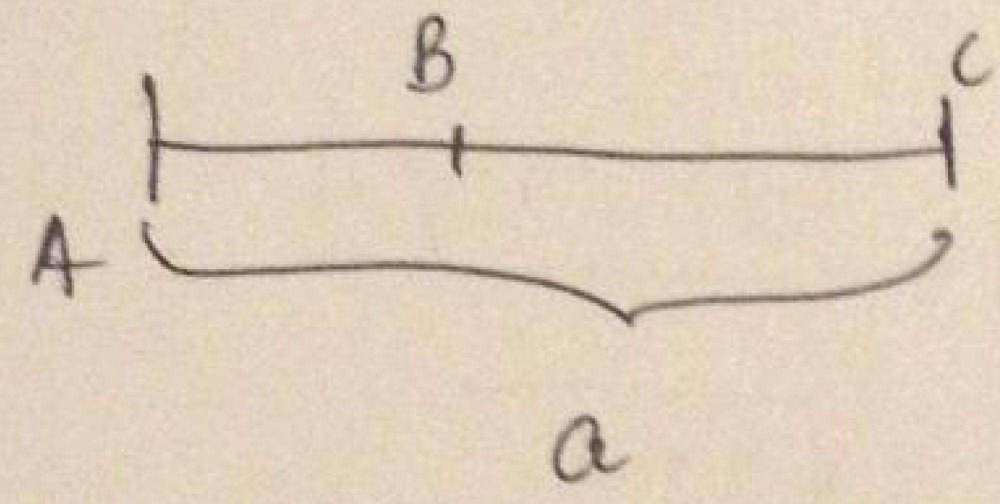
$P''=4>0 \Rightarrow$

Za $x=7$ $(y=7)$

fja $P=P_1+P_2$ je minimalna.

3) Duž dužine a podijeliti na dvije duži tako da suma trostrukog kvadrata prve duži i dvostrukog kvadrata druge duži bude minimalna. (2)

Rj



$|AC| = a$

$|AB| = x \qquad x + y = a$

$|BC| = y$

$y = a - x$

$$f = 3 \cdot x^2 + 2 \cdot y^2 \to \min$$

$$f(x) = 3x^2 + 2(a-x)^2 \to \min.$$

$$f'(x) = 0 \Rightarrow f' = 6x - 4(a-x)$$

$$6x - 4(a-x) = 0$$

$$6x - 4a + 4x = 0$$

$$10x = 4a$$

$$x = \frac{2a}{5}$$

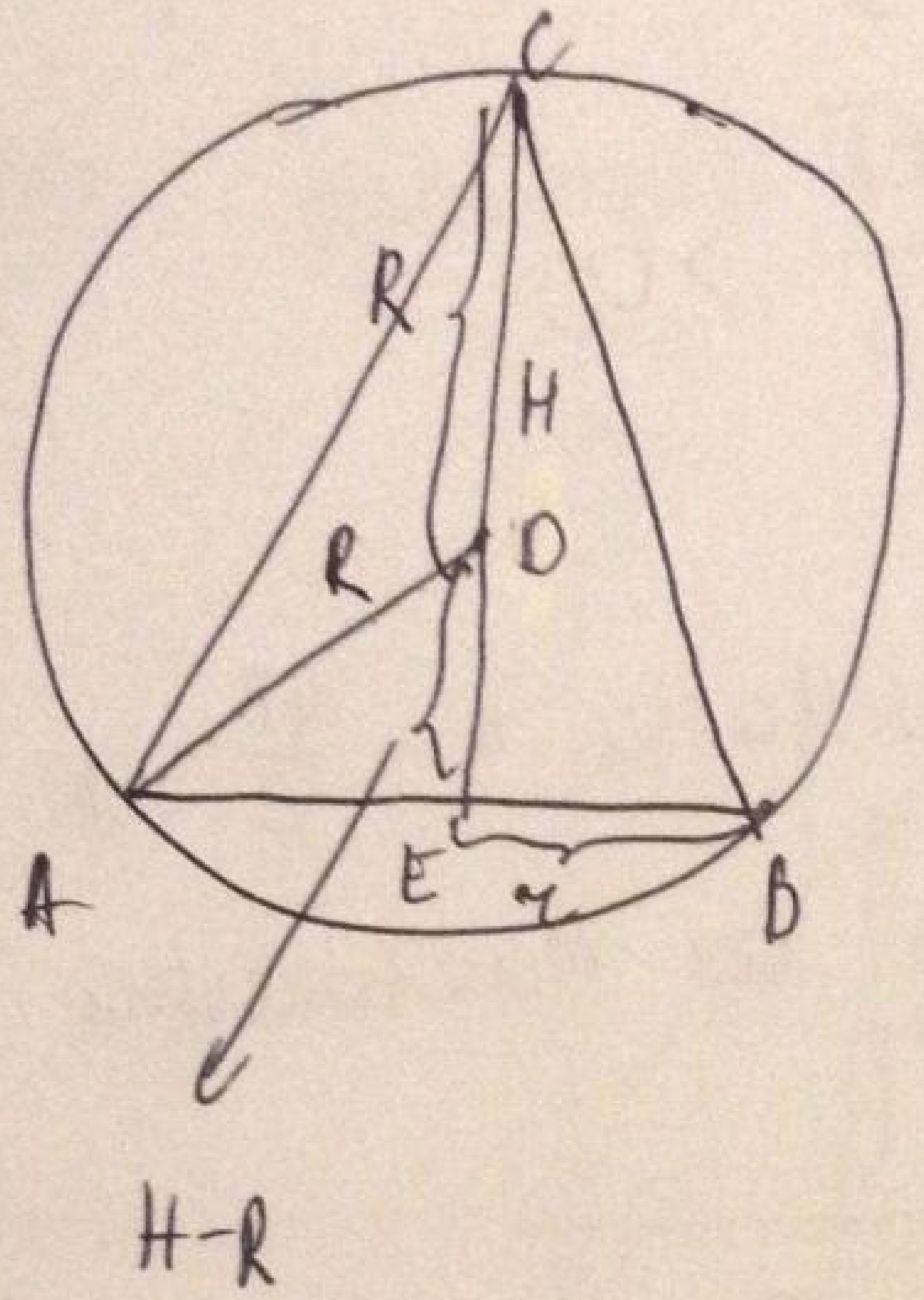
$$f'' = 6 + 4 = 10 > 0 \Rightarrow \underline{\min}$$

$$\Rightarrow |AB| = \frac{2a}{5}$$

$$|BC| = \frac{3a}{5}$$

4) Izračunati visinu kupe najveće zapremine upisane u lopti poluprečnika R.

Rj



$H = ?$

$V_{max} = ?$

$V_k = \frac{1}{3} B \cdot H = \frac{1}{3} r^2 \pi \cdot H$

$\triangle AED$:

$R^2 = r^2 + (H-R)^2$

$\cancel{R^2} = r^2 + H^2 - 2HR + \cancel{R^2}$

$r^2 = 2HR - H^2$

$r = \sqrt{2HR - H^2}$

$$V = \frac{1}{3}(2HR - H^2) H \pi = V(H)$$

$$V' = \frac{1}{3}\pi (2H^2R - H^3)' = \frac{1}{3}\pi (4RH - 3H^2)$$

$V' = 0$

$$\frac{1}{3}\pi (4RH - 3H^2) = 0$$

$$H(4R - 3H) = 0$$

$$H = 0 \quad \lor \quad H = \frac{4R}{3}$$

$(\perp)$

$$V'' = \frac{\pi}{3}(4R - 6H)$$

$$V''\left(\frac{4R}{3}\right) = \frac{\pi}{3}\left(4R - \cancel{6}^{2}\frac{4R}{\cancel{3}}\right) = \frac{\pi}{3}(-4R) = \frac{-4R\pi}{3} < 0$$

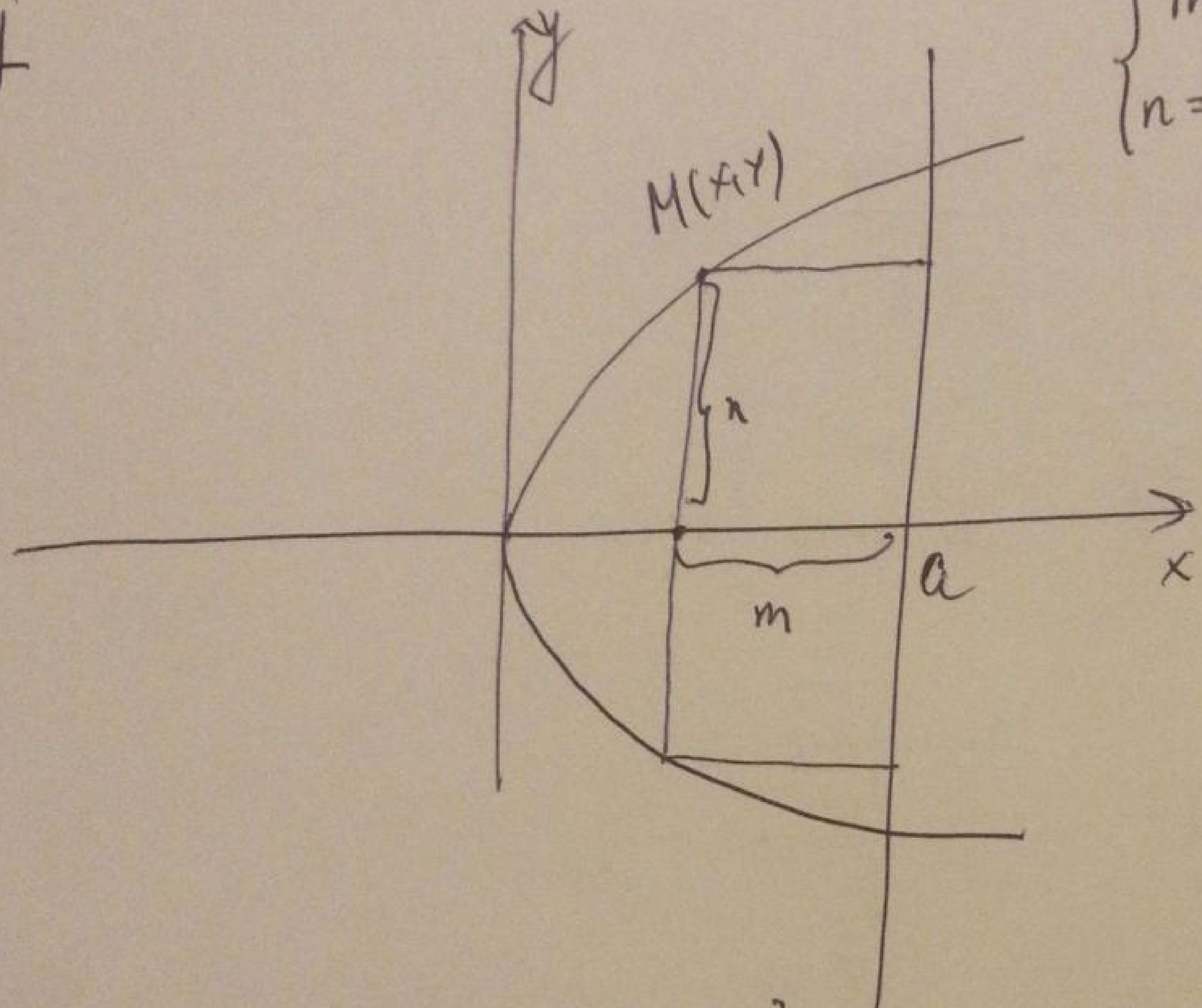
$\Rightarrow$ Drugi izvod fje zapremine u tački $H = \frac{4R}{3}$ je manji od nule $\Rightarrow H = \frac{4R}{3}$ tačka lok. maksimuma

$$V_{max} = \frac{1}{3}\pi\left(2R \cdot \frac{16R^2}{9} - \frac{4^3R^3}{3^3}\right) = \frac{\pi}{3} \cdot \left(\frac{32R^3}{9} - \frac{64R^3}{27}\right) =$$

$$= \frac{\pi}{3} \cdot \frac{32R^3}{27}$$

5) Data je parabola $y^2=2px$ i prava $x=a$, $p>0$ i $a>0$. Odrediti dimenzije pravougaonika maksimalne površine čija je jedna stranica na pravoj $x=a$ a druge dvije paralelne x-osi (dva tjemena pripadaju paraboli).

Rj



$$\begin{cases} m = a - x \\ n = y \end{cases}$$

$$P = m \cdot 2n = 2m \cdot n$$

$$P = 2 \cdot (a-x) \cdot y$$

$M(x,y) \in$ paraboli $\Rightarrow y^2 = 2px \Rightarrow \underline{x = \frac{y^2}{2p}}$

$$P = 2\left(a - \frac{y^2}{2p}\right) \cdot y = 2\left(ay - \frac{y^3}{2p}\right) = 2ay - \frac{y^3}{p}$$

$P \to max$. $\quad P' = 2a - \frac{3y^2}{p}$

$$P' = 0 \Rightarrow 2a - \frac{3y^2}{p} = 0$$

$$\frac{3y^2}{p} = 2a$$

$$3y^2 = 2ap$$

$$y^2 = \frac{2ap}{3} \qquad \underline{y = \pm\sqrt{\frac{2ap}{3}}}$$

$M(x, y)$ i $x, y > 0 \Rightarrow$

$$y = \sqrt{\frac{2ap}{3}}$$

$$P'' = -\frac{6y}{p}$$

$$P''\left(\sqrt{\frac{2ap}{3}}\right) = -\frac{6}{p}\sqrt{\frac{2ap}{3}} = \frac{-6\sqrt{2ap}}{\sqrt{3}\,p} < 0$$

$\Rightarrow$ u tački $y = \sqrt{\frac{2ap}{3}}$ P ima max.

$$P_{max}\left(\sqrt{\frac{2ap}{3}}\right) = 2\cdot\sqrt{\frac{2ap}{3}}\,a - \frac{\frac{2ap}{3}\cdot\sqrt{\frac{2ap}{3}}}{p}$$

$$P_{max} = 2a\sqrt{\frac{2ap}{3}} - \frac{2ap\sqrt{2ap}}{3\sqrt{3}\,p}$$

druga dimenzija pravoug.: $y^2 = 2px$

$$\frac{\cancel{2}a\cancel{p}}{3} = \cancel{2}\cancel{p}x$$

$$x = \frac{a}{3}$$

$$\Rightarrow \quad m = a - \frac{a}{3} = \frac{2a}{3}$$

$$2n = 2\sqrt{\frac{2ap}{3}}.$$