# Ispitivanje i crtanje grafika fje

(1) $y=\dfrac{2x^3}{x^2-4}$

1° Domen: $D_f=(-\infty,-2)\cup(-2,2)\cup(2,+\infty)$

2° Nule:

Računamo presjeke sa koordinatnim osama.

$x=0 \Rightarrow y=\dfrac{2\cdot 0^3}{0^2-4}=\dfrac{0}{-4}=0 \Rightarrow A(0,0)\in G_f$

$y=0 \Rightarrow 2x^3=0 \Rightarrow x=0 \Rightarrow$ ista tačka. (A).

3° Znak (gdje je grafik fje iznad Ox-ose $(y>0)$
gdje je grafik fje ispod Ox-ose $(y<0)$)

$y>0 \Rightarrow \dfrac{2x^3}{x^2-4}>0$

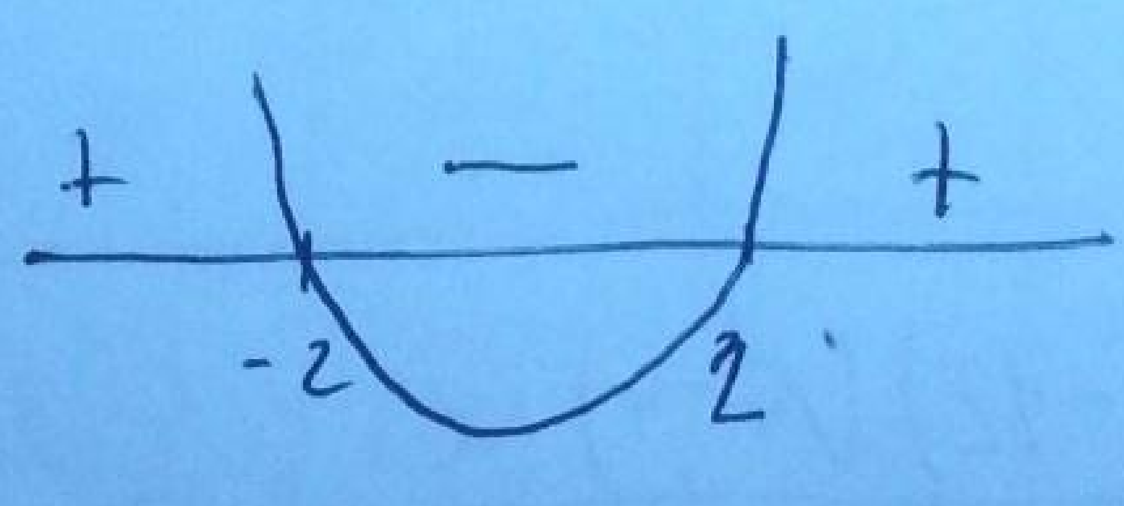
$y<0 \quad \dfrac{2x^3}{x^2-4}<0$

Ove nejednačine rješavamo tabelarno.

Fja $x^3=x^2\cdot x \Rightarrow$ Znak $x^3$ je isti kao znak fje $y=x$ jer $x^2\geq 0 \ \forall x\in\mathbb{R}$.

$x^2-4=0$

$\underline{x=\pm 2}$

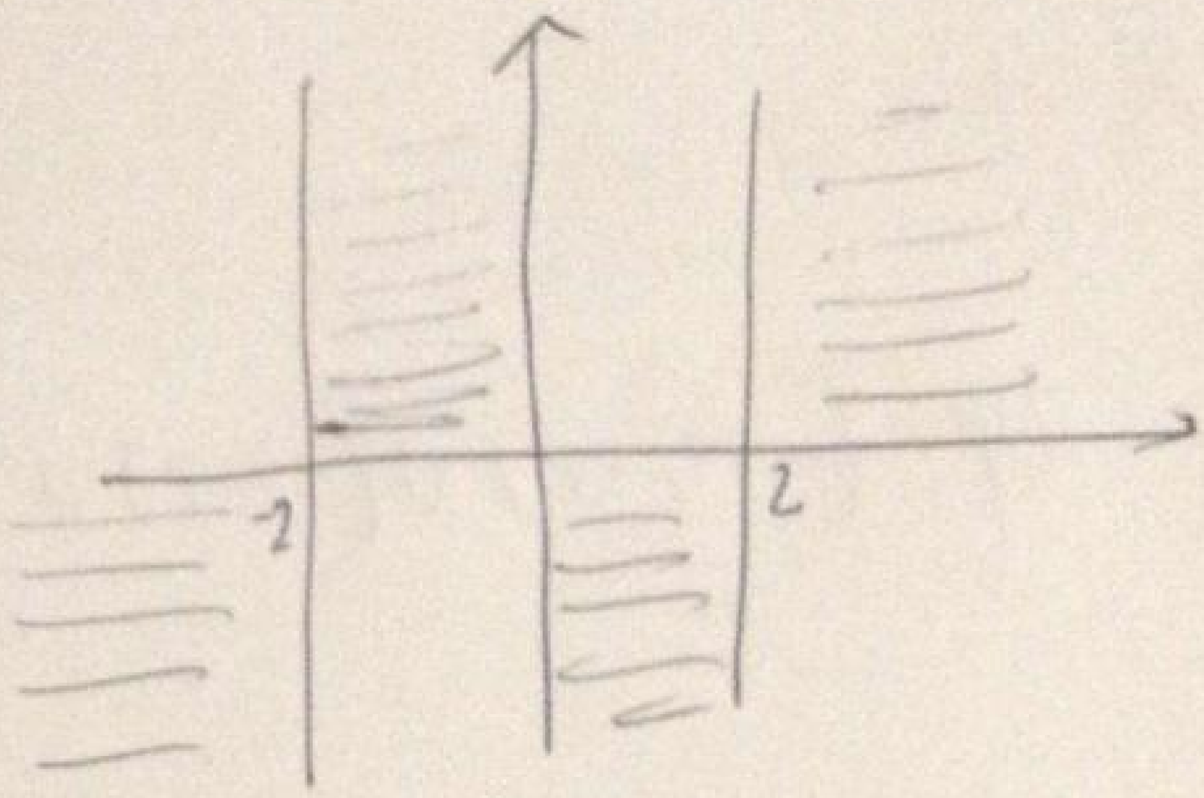



(znak fje $x^2-4$).

| $x$ | $-\infty$ … $-2$ | $-2$ … $0$ | $0$ … $2$ | $2$ … $+\infty$ |
|---|---|---|---|---|
| $x^3$ | $-$ | $-$ 0 | $+$ | $+$ |
| $x^2-4$ | $+$ 0 | $-$ | $-$ 0 | $+$ |
| $f$ | $-$ | $+$ | $-$ | $+$ |

$y > 0$ za $x \in (-2, 0) \cup (2, +\infty)$

$y < 0$ za $x \in (-\infty, -2) \cup (0, 2)$




na osjenčenim djelovima biće grafik fje

4° Konveksnost, konkavnost i prevojne tačke.

⌜Računamo $y''$, tražimo nule drugog izvoda ili tačke u kojima $y''$ nije def. Ako je $y'' > 0 \Rightarrow f \cup$ i $f'' < 0 \Rightarrow f \cap$
(konveksna) (konkavna);

ako fja u nekoj tački mijenja konv. i konkav ⇒ prevojna tačka⌟

$$y' = \frac{2x^2(x^2-12)}{(x^2-4)^2} = \frac{2x^4-24x^2}{(x^2-4)^2}$$

$$y'' = \frac{(8x^3-48x)(x^2-4)^2 - (2x^4-24x^2)2(x^2-4)\cdot 2x}{(x^2-4)^4} =$$

$$= \frac{8x(x^2-6)(x^2-4)^2 - 8x^2(x^2-12)(x^2-4)\cdot x}{(x^2-4)^4} =$$

$$= \frac{8x(x^2-4)\left((x^2-6)(x^2-4)-x^2(x^2-12)\right)}{(x^2-4)^4} =$$

$$= \frac{8x(x^2-4)(\cancel{x^4}-10x^2+24-\cancel{x^4}+12x^2)}{(x^2-4)^4} = \frac{8x(x^2-4)(2x^2+24)}{(x^2-4)^4} =$$

$$= \frac{16x(x^2-4)(x^2+12)}{(x^2-4)^4} .$$

$$y''=0 \Rightarrow \underline{x=0} \ \vee \ \underline{x=\pm 2} \ (\notin D)$$

$$\left.\begin{array}{l} y''>0 \Leftrightarrow \dfrac{16x(x^2-4)(x^2+12)}{(x^2-4)^4}>0 \\[2ex] y''<0 \Leftrightarrow \dfrac{16x(x^2-4)(x^2+12)}{(x^2-4)^4}<0 \end{array}\right\} \quad \begin{array}{l} (\forall x\in D_f) \\[1ex] \dfrac{16(x^2+12)}{(x^2-4)^4}>0 \Rightarrow \end{array}$$

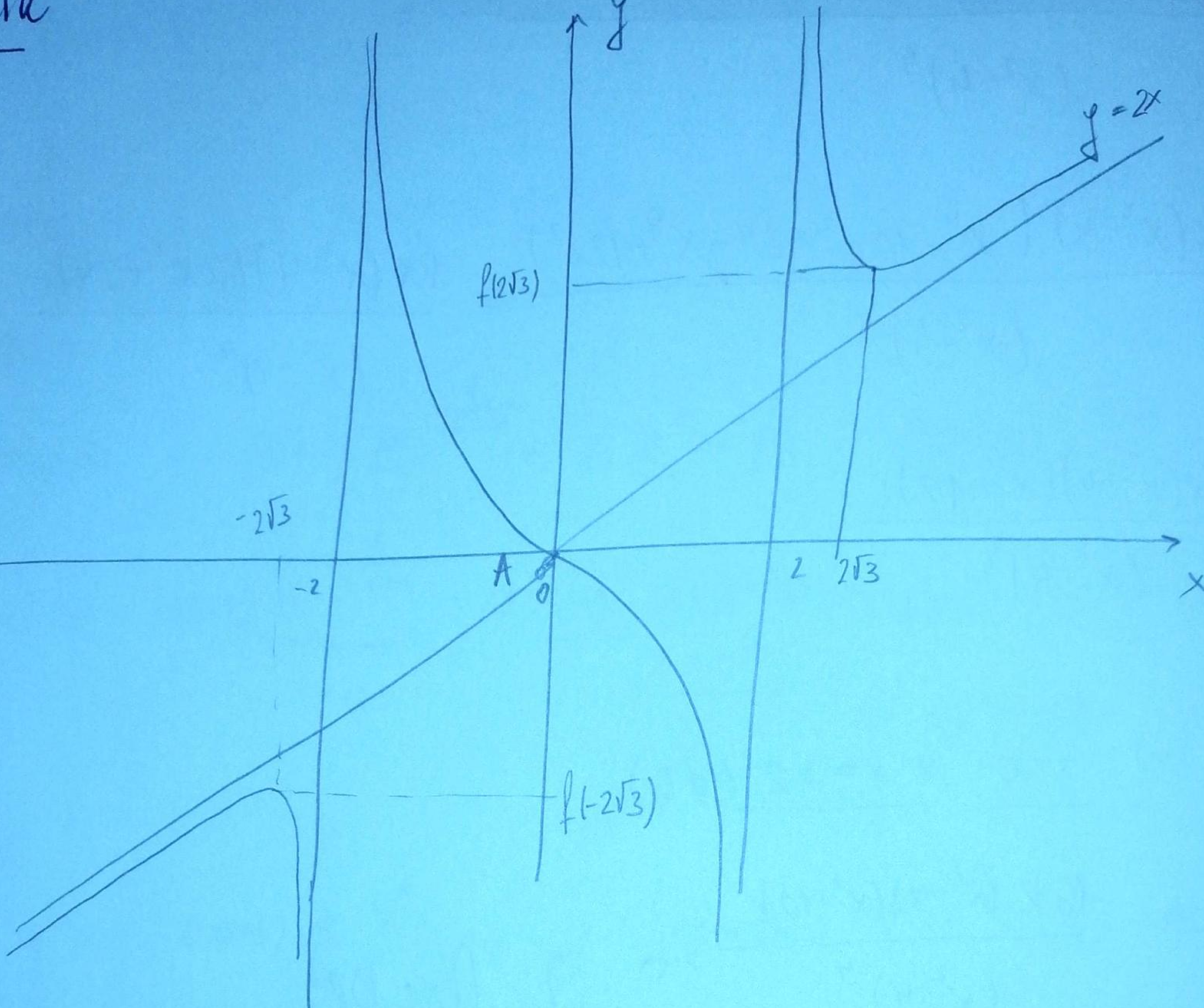
Znak $y''$ zavisi od $x\cdot(x^2-4)$. (isto kao za znak).

| | $-\infty$ … $-2$ | $-2$ … $0$ | $0$ … $2$ | $2$ … $+\infty$ |
|---|---|---|---|---|
| $x$ | $-$ | $-$ | 0 $+$ | $+$ |
| $x^2-4$ | $+$ | 0 $-$ | $-$ | 0 $+$ |
| $y''$ | $-$ | $+$ | $-$ | $+$ |
| $y$ | $\cap$ | $\cup$ | $\cap$ | $\cup$ |

$\Rightarrow$ Za $x=0 \Rightarrow$

$y(0)=0$ $\quad A(0,0)$ prev. t.

grafik



③ $y=(x+1)e^{\frac{1}{x+1}}$



1° Domen: $D_f=(-\infty,-1)\cup(-1,+\infty)$

2° Nule:

$x=0 \Rightarrow y=(0+1)e^{\frac{1}{0+1}} = 1\cdot e^{1} = e \Rightarrow A(0,e)\in G_f$

$y=0 \Leftrightarrow (x+1)\underbrace{e^{\frac{1}{x+1}}}_{>0}=0 \Rightarrow x+1=0$

$\underbrace{x=-1}$

$B(-1,0)\notin G_f$ jer $x=-1\notin D$.

3° Znak:

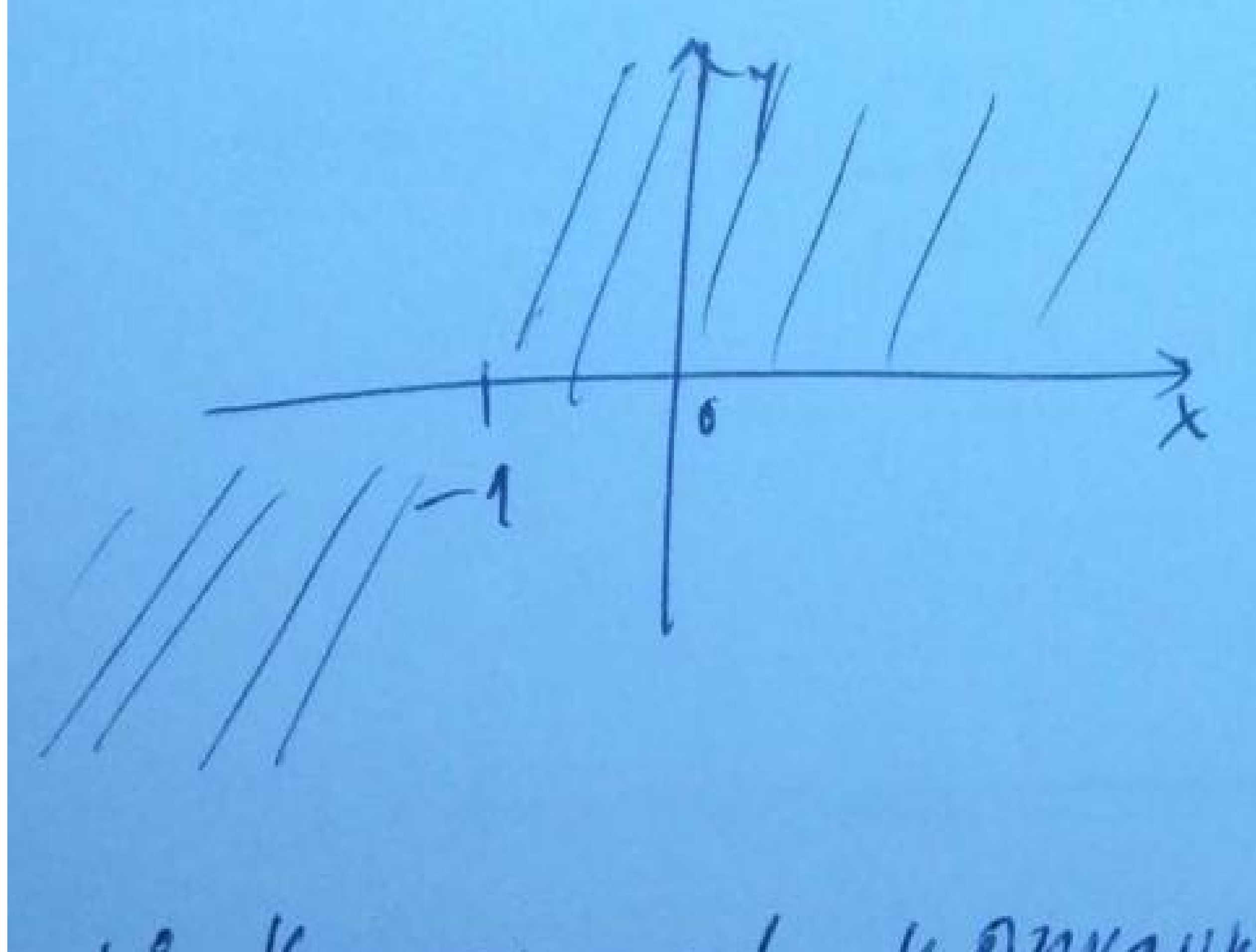
$y>0 \Leftrightarrow (x+1)e^{\frac{1}{x+1}}>0$

$y<0 \Leftrightarrow (x+1)e^{\frac{1}{x+1}}<0$

$(\forall x\in D_f)\ e^{\frac{1}{x+1}}>0 \rightarrow$ znak $y$ zavisi od $x+1$.

$y>0 \Leftrightarrow x+1>0$, $\underbrace{x>-1}$

$y<0$ za $\underbrace{x<-1}$




na osjenčenom dijelu će se pojaviti f-ja

4° Konveksnost, konkavnost i prevojne tačke

$$y''=\left(\frac{x}{x+1}e^{\frac{1}{x+1}}\right)'=\frac{x+1-x}{(x+1)^2}e^{\frac{1}{x+1}}+\frac{x}{x+1}\cdot e^{\frac{1}{x+1}}\cdot\left(-\frac{1}{(x+1)^2}\right)=$$

$$=\frac{1}{(x+1)^2}e^{\frac{1}{x+1}}\left(1-\frac{x}{x+1}\right)=$$

$$= \frac{1}{(x+1)^2} e^{\frac{1}{x+1}} \cdot \frac{\cancel{x}+1-\cancel{x}}{x+1} = \underline{\frac{1}{(x+1)^2(x+1)} e^{\frac{1}{x+1}}}$$

$y'' \neq 0 \quad \forall x \in D_f$. $\quad \left[ (x+1)^2 \geq 0 \text{ i } e^{\frac{1}{x+1}} > 0 \right]$

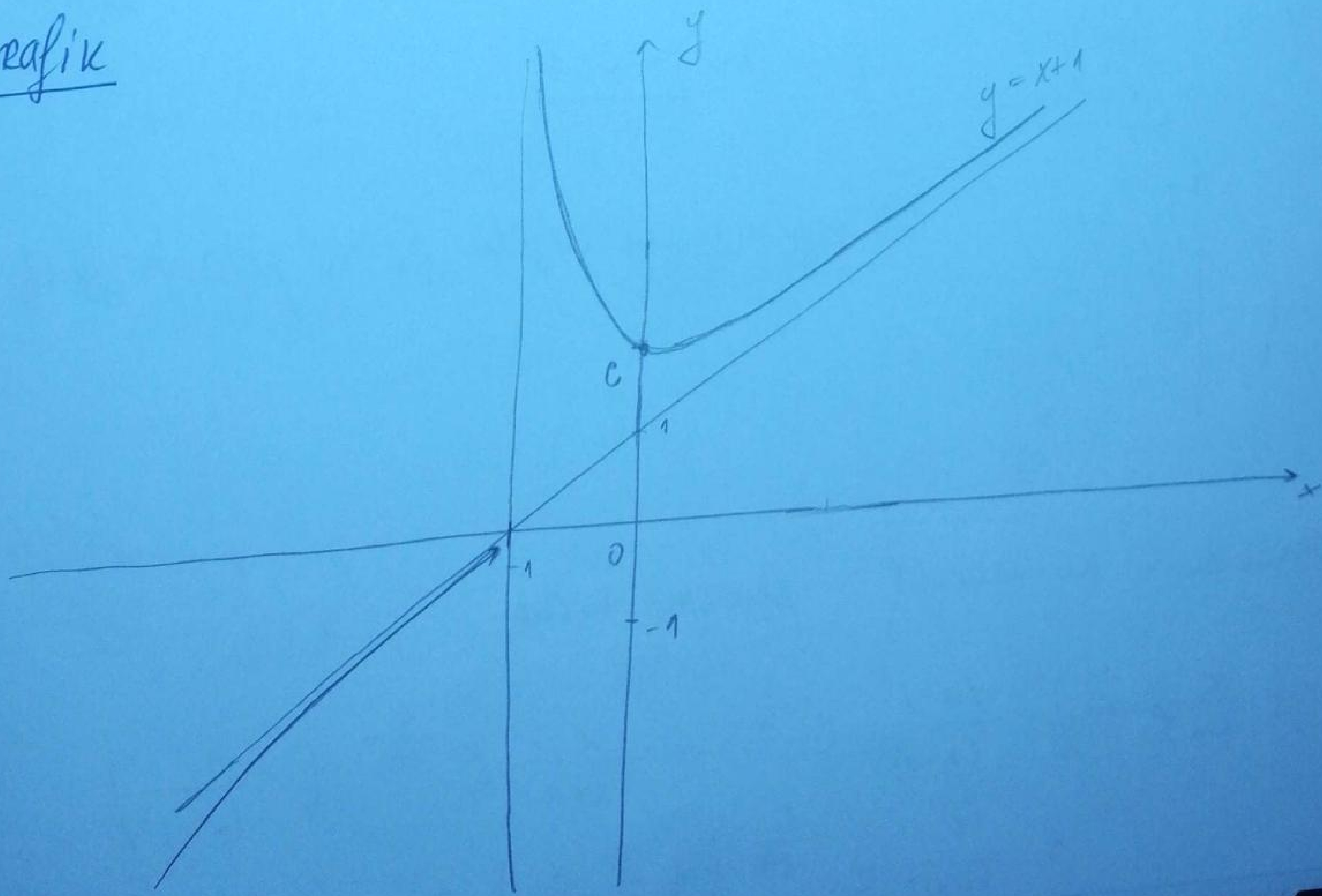
$y'' > 0$ za $x+1 > 0 \Rightarrow x > -1$

$y'' < 0$ za $x+1 < 0 \Rightarrow x < -1$

| $x$ | $-\infty$ | $-1$ | $+\infty$ |
|---|---|---|---|
| $y''$ | $-$ | $+$ | |
| $y$ | $\cap$ | $\cup$ | |

$x=-1$ nije prev. tacka jer $x=-1 \notin D_f$.

<u>grafik</u>



⑤ $y = \ln \frac{x+3}{1-x}$

4

1° Domen: $D_f = (-3, 1)$

2° Nule:

$x=0 \Rightarrow y = \ln \frac{0+3}{1-0} = \ln \frac{3}{1} = \ln 3 \Rightarrow A(0, \ln 3) \in G_f.$

$y=0 \Rightarrow 0 = \ln \frac{x+3}{1-x}$

$\frac{x+3}{1-x} = 1$  $B(-1, 0) \in G_f$

$x+3 = 1-x$

$2x = -2$

$\underline{x = -1}$

3° Znak

$$\left.\begin{array}{l} y>0 \Leftrightarrow \ln \frac{x+3}{1-x} > 0 \\ y<0 \Leftrightarrow \ln \frac{x+3}{1-x} < 0 \end{array}\right\} \Rightarrow$$

$\ln \frac{x+3}{1-x} > 0$  $\ln \frac{x+3}{1-x} < 0$

$\frac{x+3}{1-x} > 1$  $\frac{x+3}{1-x} < 1$

$\frac{x+3}{1-x} - 1 > 0$  $\frac{x+3}{1-x} - 1 < 0$

$\frac{x+3-1+x}{1-x} > 0$  $\frac{x+3-1+x}{1-x} < 0$

$\frac{2x+2}{1-x} > 0$  $\frac{2x+2}{1-x} < 0$

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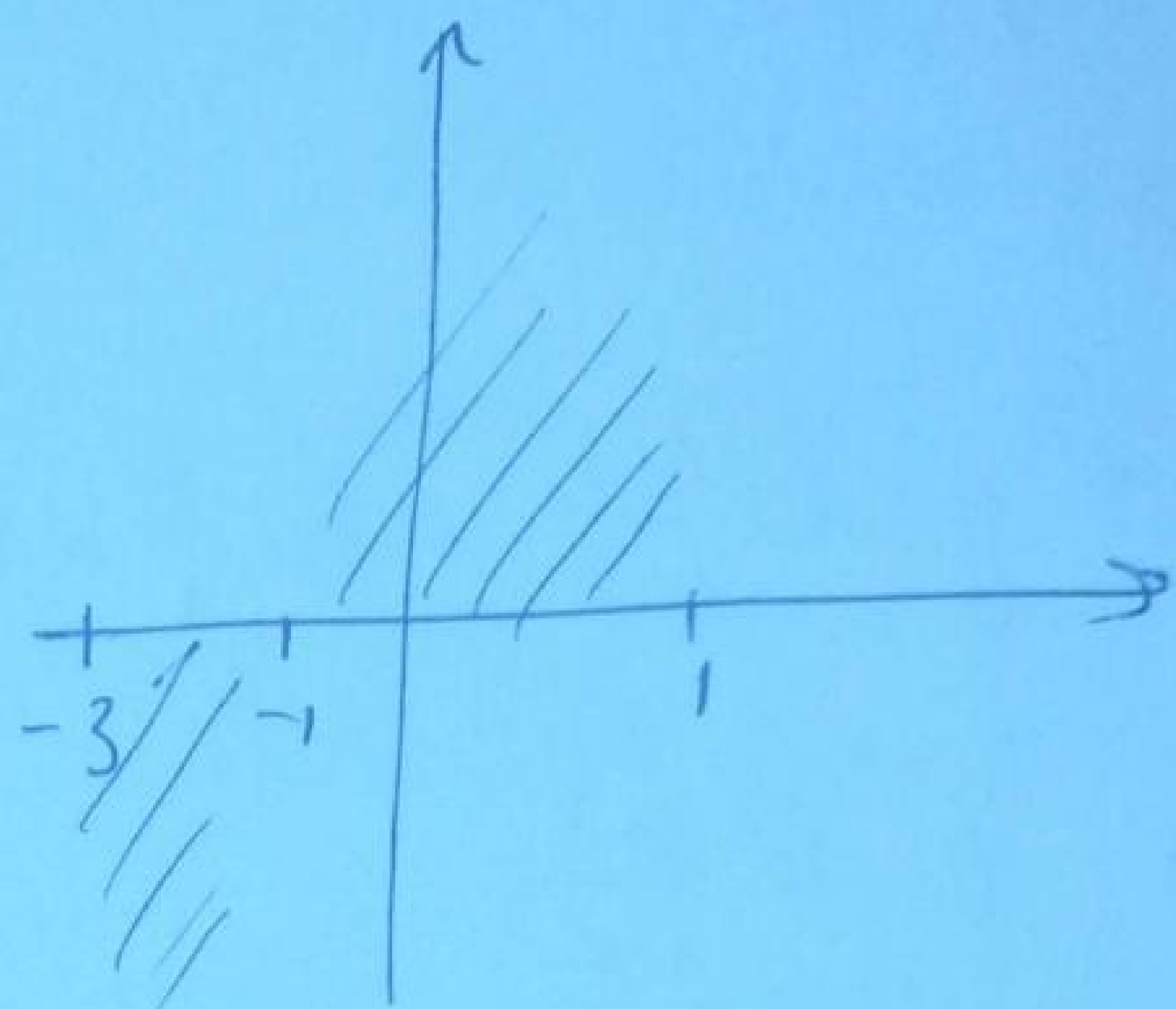
$2x+2=0$  $1-x=0$

$\underline{x=-1}$  $\underline{x=1}$

| $x$ | $-\infty$ | $-3$ | $-1$ | $1$ | $+\infty$ |
|---|---|---|---|---|---|
| $2x+2$ | $-$ | $-$ | $0$ $+$ | $+$ | |
| $1-x$ | $+$ | $+$ | $+$ | $0$ $-$ | |
| $f$ | $-$ | $-$ | $+$ | $-$ | |

domen →



$f > 0$ za $x \in (-1, 1)$

$f < 0$ za $x \in (-3, -1)$

na osjenčenom dijelu biće grafik fje

4° Konv, konkav i prev. tačke

$$y'' = \left(\frac{4}{(x+3)(1-x)}\right)' = -\frac{4}{(x+3)^2(1-x)^2}\left(1-x+(x+3)(-1)\right) =$$

$$= \frac{-4}{(x+3)^2(1-x)^2}(1-x-x-3) = \frac{-4(-2x-2)}{(x+3)^2(1-x)^2} = \frac{8(x+1)}{(x+3)^2(1-x)^2}$$

$y'' = 0$ za $\underline{x = -1}$

$$\left.\begin{aligned} y'' > 0 &\Leftrightarrow \frac{8(x+1)}{(x+3)^2(1-x)^2} > 0 \\ y'' < 0 &\Leftrightarrow \frac{8(x+1)}{(x+3)^2(1-x)^2} < 0 \end{aligned}\right\}$$

$(\forall x \in D_f)$ $\frac{8}{(x+3)^2(1-x)^2} > 0 \rightarrow$

Znak $y''$ zavisi od $x+1$.

$y'' > 0 \Leftrightarrow x+1>0$ $\qquad$ $y''<0$ za $\underline{x<-1}$ $\qquad$ 5

$\underline{x>-1}$

| $x$ | $-\infty$ | $-3$ | $-1$ | $1$ | $+\infty$ |
|---|---|---|---|---|---|
| $y''$ | $-$ | $-$ | $0$ | $+$ | $+$ |
| $y$ | | $\cap$ | | $\cup$ | |

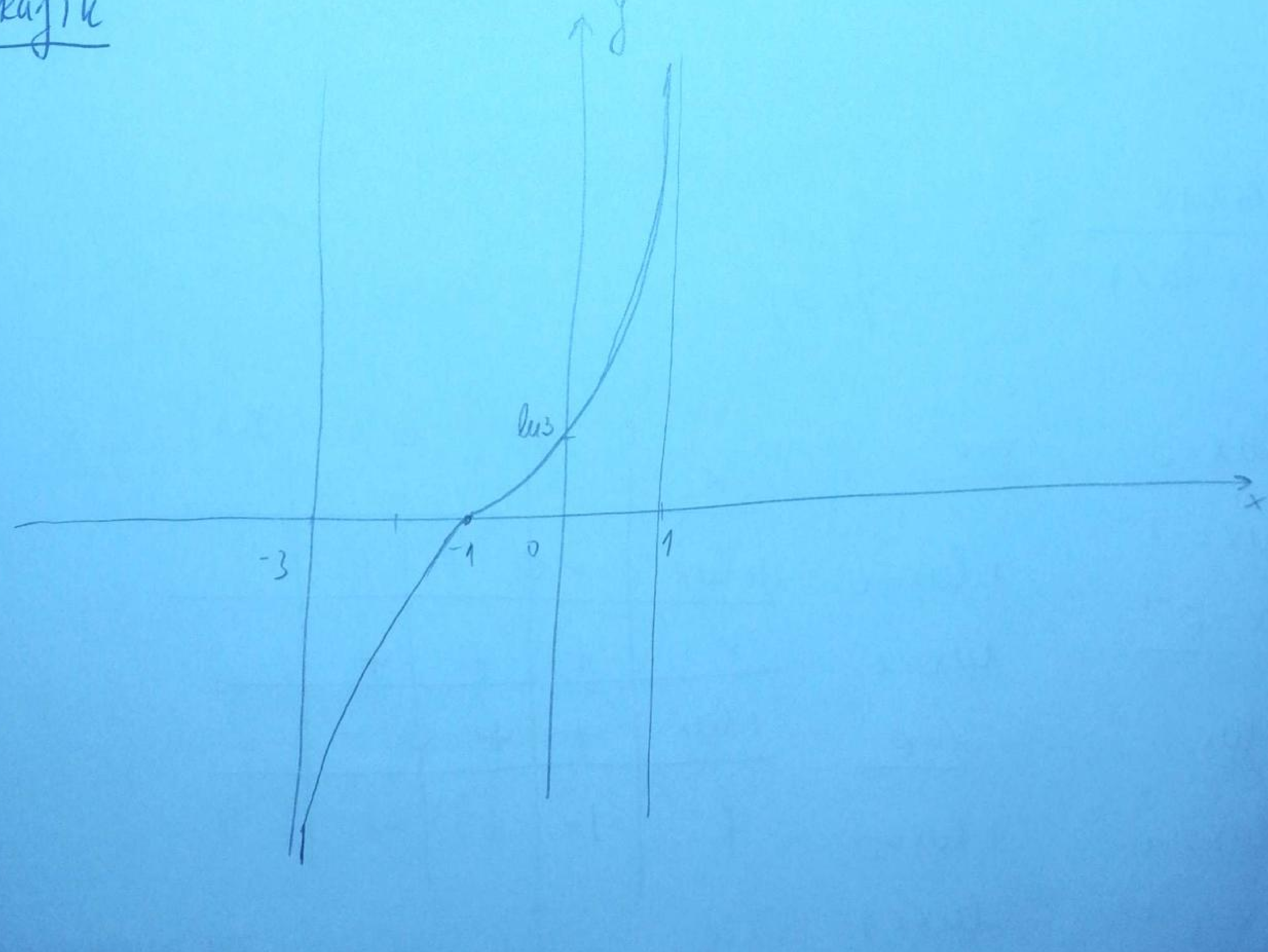
$\underbrace{\qquad\qquad}_{\text{domen}}$

Na domeni postoji jedna prev. tačka $x=-1$

$f(-1) = \ln\frac{2}{2} = 0$

$T_p(-1, 0)$.

grafik



⑦ $y = \dfrac{1+\ln x}{x(1-\ln x)}$

1° $D_f = (0, e) \cup (e, +\infty)$

2° Nule:

$x = 0 \notin D$ (ne možemo tražiti presjek sa $Oy$-osom)

$y = 0 \Leftrightarrow 1+\ln x = 0$

$\ln x = -1$

$\underline{x = e^{-1}}$

$\left(\frac{1}{e}, 0\right) \in G_f$.

3° Znak:

$\dfrac{1+\ln x}{x(1-\ln x)} \gtrless 0 \quad \left(\begin{matrix} y>0 \\ y<0 \end{matrix}\right)$

$1+\ln x = 0$

$\ln x = -1$

$\underline{x = e^{-1}}$

$1+\ln x > 0$

$\ln x > -1$

$\underline{x > e^{-1}}$

$\underline{x = 0}$

$1-\ln x = 0$

$\ln x = 1$

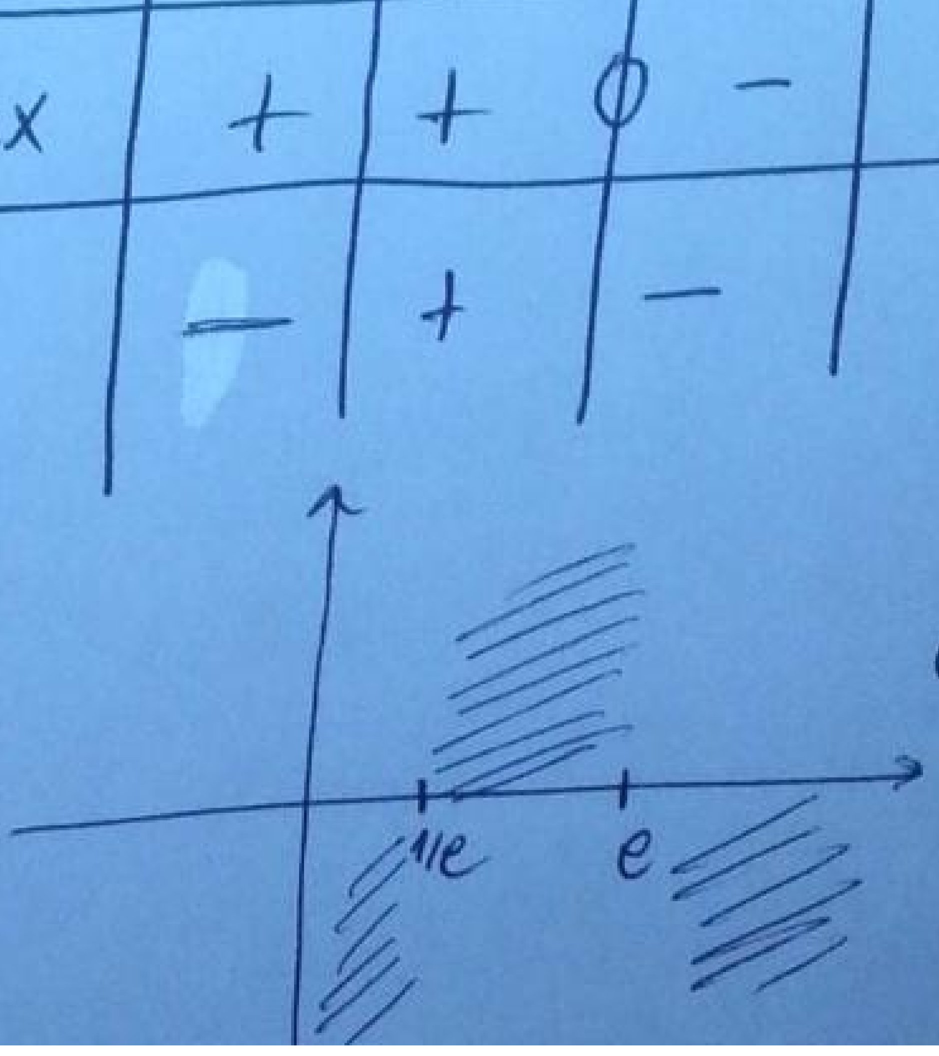
$\underline{x = e}$

$1-\ln x > 0$

$\ln x < 1$

$\underline{x < e}$

| $x$ | $0$ … $1/e$ | $1/e$ … $e$ | $e$ … $+\infty$ |
|---|---|---|---|
| $1+\ln x$ | $-$ 0 | $+$ | $+$ |
| $x$ | $+$ | $+$ | $+$ |
| $1-\ln x$ | $+$ | $+$ 0 | $-$ |
| $f$ | $-$ | $+$ | $-$ |




na osjenčenim dijelovima će se pojaviti grafik fje

4° Konveksnost, konkavnost i prevojne tačke:

$$y'' = \left(\frac{\ln^2 x + 1}{x^2(1-\ln x)^2}\right)' = \frac{2\ln x \cdot \frac{1}{\cancel{x}}\left(x^{\cancel{2}}(1-\ln x)^2\right) - (\ln^2 x+1)\left(2x(1-\ln x)^2 + x^{\cancel{2}} \cdot 2(1-\ln x)\cdot\left(-\frac{1}{\cancel{x}}\right)\right)}{x^4(1-\ln x)^4} =$$

$$= \frac{2x\ln x(1-\ln x)^2 - (\ln^2 x+1)\left(\overline{2x(1-\ln x)^2} - \overline{2x(1-\ln x)}\right)}{x^4(1-\ln x)^4} =$$

$$= \frac{2x\ln x(1-\ln x)^2 - (\ln^2 x+1)\cdot 2x(1-\ln x)(\cancel{1}-\ln x-\cancel{1})}{x^4(1-\ln x)^4} =$$

$$= \frac{\overbrace{2x\ln x(1-\ln x)^2} + \overbrace{2x\ln x(1-\ln x)(\ln^2 x+1)}}{x^4(1-\ln x)^4} =$$

$$= \frac{2x\ln x(1-\ln x)\left[1-\ln x+\ln^2 x+1\right]}{x^4(1-\ln x)^4} =$$

$$= \frac{2x\ln x(1-\ln x)(\ln^2 x-\ln x+2)}{x^4(1-\ln x)^4}.$$

$y''=0 \Leftrightarrow$ $\underline{x=0 \notin D_f}$ $\vee$ $\ln x = 0$, $\underline{x=1}$

$1-\ln x=0$

$\ln x = 1$

$\underline{x=e\notin D_f}$

$\ln^2 x - \ln x + 2 = 0$

$\ln x = t$

$t^2 - t + 2 = 0 \quad \notin R$

$t_{1,2} = \frac{1\pm\sqrt{1-8}}{2}$

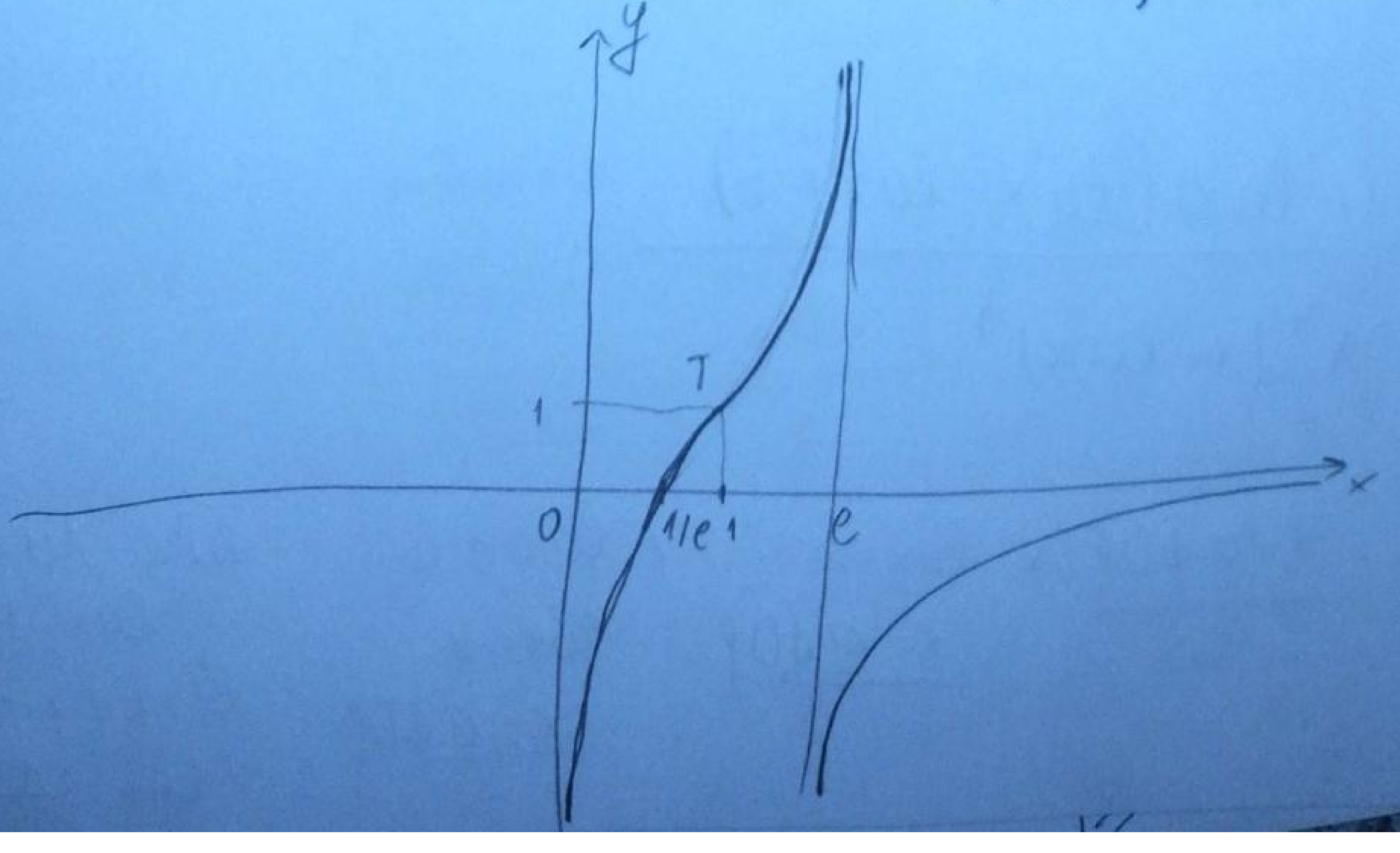
$\ln^2 x - \ln x + 2 > 0 \quad \forall x \in D_f$ jer $\left( ax^2+bx+c>0 \ \forall x \right.$ ako $a>0$, $\left. D<0 \right)$

Znak $y''$ ne zavisi od $\dfrac{\ln^2 x - \ln x + 2}{x^4(1-\ln x)^4} > 0$

Znak $y''$ zavisi od $x \cdot \ln x (1-\ln x)$.

| $x$ | $0$ – $1/e$ | $1/e$ – $1$ | $1$ – $e$ | $e$ – $+\infty$ |
|---|---|---|---|---|
| $x$ | + | + | + | + |
| $\ln x$ | − | − | 0 + | + |
| $1-\ln x$ | + | + | + 0 | − |
| $y''$ | − | − | + | − |
| $y$ | $\cap$ | $\cap$ | $\cup$ | $\cap$ |

Prevojna tačka za $x=1 \rightarrow f(1) = \dfrac{1+\ln 1}{1(1-\ln 1)} = \dfrac{1}{1} = 1 \quad T_p(1,1)$



② $y=\sqrt[3]{x^3+3x^2}$

1) $D_f=R$

2) $f(-x)=\sqrt[3]{(-x)^3+3x^2}=\sqrt[3]{-x^3+3x^2}\neq f(x)$
$\neq -f(x)$

Ни парна, ни непарна.

3) Нуле и знак:

$f(x)=0 \Leftrightarrow \sqrt[3]{x^3+3x^2}=0 \Leftrightarrow x^3+3x^2=0$

$x^2(x+3)=0$

$x^2=0,\ x+3=0$

$x=-3$

$N_1(0,0)$

$N_2(-3,0)$

$f(x)>0 \Leftrightarrow \sqrt[3]{x^3+3x^2}>0 \Leftrightarrow x^3+3x^2>0$

$x^2(x+3)>0$

$\underbrace{}_{\geq 0}\quad x>-3$

$f(x)<0 \Leftrightarrow \sqrt[3]{x^3+3x^2}<0 \Leftrightarrow x^3+3x^2<0$

$x+3<0$

$x<-3$

$f(x)>0$ за $x\in(-3,+\infty)$, а $f(x)<0$ за $x\in(-\infty,-3)$

4) Асимптоте

$\lim_{x\to\pm\infty} f(x)=\pm\infty$

$y=x+1,\ x\to\pm\infty$

5) $T_{min}(0,0)$, $T_{max}(-2,\sqrt[3]{4})$

6) $y''=\left(\frac{x+2}{\sqrt[3]{x(x+3)^2}}\right)'=\frac{\sqrt[3]{x(x+3)^2}-(x+2)\cdot\frac{1}{3}\cdot(x(x+3)^2)^{-\frac{2}{3}}\cdot((x+3)^2+x\cdot 2(x+3))}{\sqrt[3]{x(x+3)^2}^2}$

$=\frac{3\cdot\sqrt[3]{x(x+3)^2}-(x+2)\cdot\frac{1}{\sqrt[3]{x^2(x+3)^4}}\cdot(x+3)(3x+3)}{3\cdot\sqrt[3]{x^2(x+3)^4}}$

$=\frac{3\sqrt[3]{x^3\cdot(x+3)^6}-3(x+2)(x+3)(x+1)}{3\cdot\left(\sqrt[3]{x^2(x+3)^4}\right)^2}=\frac{3x(x+3)^2-3(x+1)(x+2)(x+3)}{3\cdot x^{\frac{4}{3}}\cdot(x+3)^{\frac{8}{3}}}=$

$$= \frac{\cancel{3}(x+3)\left(x(x+3)-(x+1)(x+2)\right)}{\cancel{3}\cdot x^{\frac{4}{3}}\cdot(x+3)^{\frac{8}{3}}} =$$

$$= \frac{\cancel{x^2}+\cancel{3x}-\cancel{x^2}-\cancel{3x}-2}{x^{\frac{4}{3}}\cdot(x+3)^{\frac{5}{3}}} = -\frac{2}{x^{\frac{4}{3}}\cdot(x+3)^{\frac{5}{3}}} = \frac{-2}{\sqrt[3]{x^4}\cdot\sqrt[3]{(x+3)^5}}$$

$y''=0 \Leftrightarrow -2=0$ (немогуће).

$$y''>0 \Leftrightarrow \frac{-2}{\sqrt[3]{x^4}\cdot\sqrt[3]{(x+3)^5}}>0 \Leftrightarrow$$

$$\frac{1}{\sqrt[3]{x^4}\cdot\sqrt[3]{(x+3)^5}}<0, \text{ иј}$$

$$x+3<0$$

$$\underbrace{x<-3}$$

$y''<0 \Leftrightarrow x>-3$

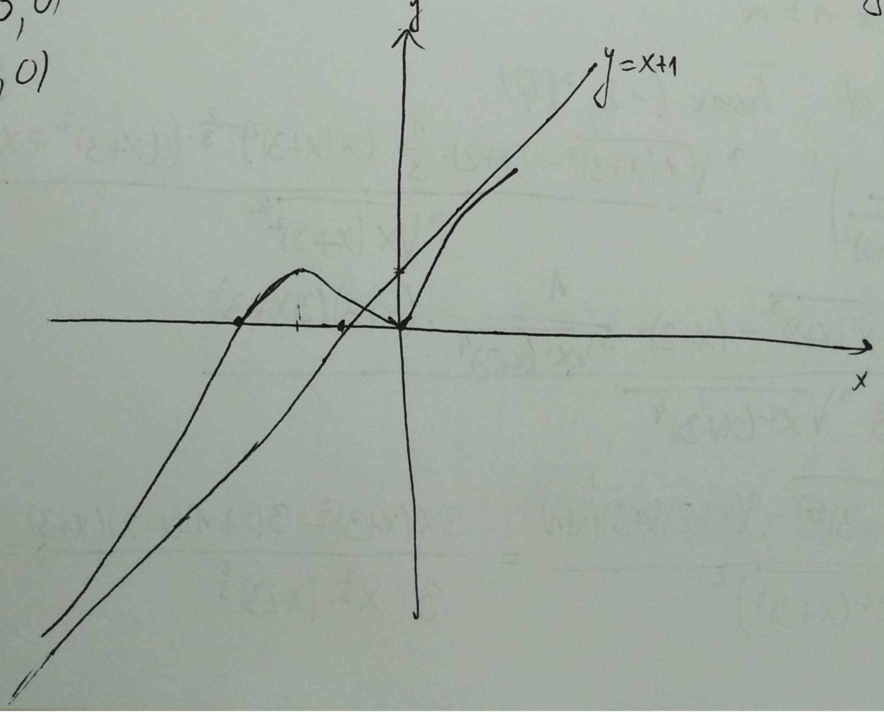
| | $-\infty$ | $-3$ | | $0$ | $+\infty$ |
|---|---|---|---|---|---|
| $y''$ | $+$ | | $-$ | | $-$ |
| $y$ | $\cup$ | | $\cap$ | | $\cap$ |

превојна превојна

$P_1(-3,0)$

$P_2(0,0)$

$y=x+1$



④ $y = \frac{e^x}{x\sqrt{1-x}}$

1) $D_f = (-\infty, 0) \cup (0,1)$

2) $f(-x) = \frac{e^{-x}}{-x\sqrt{1+x}} \neq f(x)$
$\neq -f(x)$

Ни парна, ни непарна.

3) Нуле и знак

$f(x) = 0 \Leftrightarrow \frac{e^x}{x\sqrt{1-x}} = 0 \Rightarrow e^x = 0 \ (\perp)$

Нема нула.

$f(x) > 0 \Leftrightarrow \frac{\overset{>0}{e^x}}{x\underset{>0}{\sqrt{1-x}}} > 0 \Leftrightarrow x > 0$

Дакле, $f(x) > 0 \Leftrightarrow x \in (0,1)$

$f(x) < 0 \Leftrightarrow \frac{\overset{>0}{e^x}}{x\underset{>0}{\sqrt{1-x}}} < 0 \Leftrightarrow x < 0$

Дакле, $f(x) < 0 \Leftrightarrow x \in (-\infty, 0)$.

4) Асимптоте.

$\lim_{x\to 0^+} f(x) = +\infty$

$\lim_{x\to 0^-} \frac{e^x}{x\sqrt{1-x}} = -\infty$

$\lim_{x\to 1-0} f(x) = +\infty$

$x=0$, $x=1$ В.А

$\lim_{x\to -\infty} f(x) = 0$

$y=0$ Х.А

5) $T_{min}(\frac{1}{2}, 2\sqrt{2}\sqrt{e})$

⑥ $f''(x) = \left(\dfrac{e^x(2x^2-5x+2)}{2x^2(1-x)^{\frac{3}{2}}}\right)' =$

$$\frac{\left(e^x(2x^2-5x+2)+e^x(4x-5)\right)\left(2x^2(1-x)^{\frac{3}{2}}\right)-\left(e^x(2x^2-5x+2)\right)\cdot\left(4x(1-x)^{\frac{3}{2}}+2x^2\cdot\frac{3}{2}(1-x)^{\frac{1}{2}}(-1)\right)}{\left(2x^2(1-x)^{\frac{3}{2}}\right)^2}$$

$$\frac{e^x(2x^2-x-3)\cdot 2x^2(1-x)^{\frac{3}{2}}-e^x(2x^2-5x+2)\cdot\left(4x(1-x)^{\frac{3}{2}}-3x^2(1-x)^{\frac{1}{2}}\right)}{\left(2x^2(1-x)^{\frac{3}{2}}\right)^2}$$

$\ulcorner\; 4x(1-x)^{\frac{3}{2}}-3x^2(1-x)^{\frac{1}{2}} = x(1-x)^{\frac{1}{2}}(4\cdot(1-x)-3\cdot x) = x(1-x)^{\frac{1}{2}}(4-7x) \;\lrcorner$

$$=\frac{2e^x\cdot(2x^2-x-3)x^2(1-x)^{\frac{3}{2}}-e^x(2x^2-5x+2)\cdot x(1-x)^{\frac{1}{2}}\cdot(4-7x)}{\left(2x^2(1-x)^{\frac{3}{2}}\right)^2}$$

$$=\frac{e^x\cdot x(1-x)^{\frac{1}{2}}\left(2(2x^2-x-3)\cdot x(1-x)-(2x^2-5x+2)(4-7x)\right)}{4x^4\cdot(1-x)^3}$$

$$\frac{e^x\cdot x\cdot(1-x)^{\frac{1}{2}}\left((2x^2-x-3)\cdot(2x-2x^2)-(2x^2-5x+2)(4-7x)\right)}{4x^4(1-x)^3}$$

$$\frac{e^x\cdot x\cdot(1-x)^{\frac{1}{2}}\left(4x^3-4x^4-2x^2+2x^3-6x+6x^2-8x^2+14x^3+20x-35x^2-8+14x\right)}{4\cdot x^4\cdot(1-x)^3}$$

$$=\frac{e^x\cdot(-4x^4+20x^3-39x^2+28x-8)}{4\cdot(1-x)^{\frac{5}{2}}\cdot x^3}=-\frac{e^x(4x^4-20x^3+39x^2-28x+8)}{4(1-x)^{\frac{5}{2}}\cdot x^3}$$

$4x^4-20x^3+39x^2-28x+8=(4x^4+25x^2+4-20x^3-20x+8x^2)+6x^2-8x+4=$

$=(2x^2-5x+2)^2+4x^2-8x+4+2x^2=$

$=(2x^2-5x+2)^2+(2x-2)^2+2x^2\geq 0$

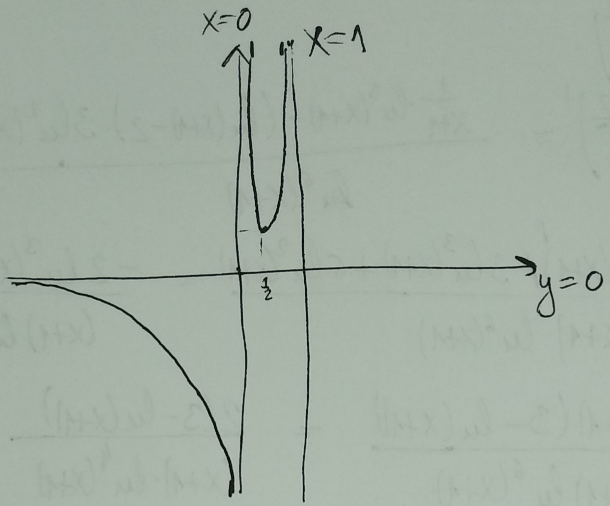
$e^x>0 \quad \forall x$

$1-x>0 \quad \forall x\in D_f$

$f''(x)>0 \Leftrightarrow x^3>0 \Leftrightarrow x>0$

$f''(x)<0 \Leftrightarrow x<0$

| $x$ | $-\infty$ | $0$ | | $1$ | $+\infty$ |
|---|---|---|---|---|---|
| $f''(x)$ | $-$ | | $+$ | | |
| $f(x)$ | $\frown$ | | $\smile$ | | |




⑥ $y = \dfrac{x+1}{\ln^2(x+1)}$

1) $D_f = (-1,0) \cup (0,+\infty)$

2) Пошто домен није симетричан, ф-ја није парна, нити непарна.

3) Нуле и знак.

$f(x) = 0 \iff \dfrac{x+1}{\ln^2(x+1)} = 0 \iff x = -1$

$\underline{N(-1,0) \notin D_f}$ $\Rightarrow$ Нема нула ф-је.

$f(x) > 0 \iff \dfrac{\overset{>0}{(x+1)}}{\underset{>0}{\ln^2(x+1)}} > 0$

$\underline{f(x) > 0 \quad \forall x \in D_f}$

4) Асимптоте

$\lim\limits_{x \to -1+0} f(x) = 0$

$\lim\limits_{x \to 0-} f(x) = +\infty$

$\lim\limits_{x \to 0+} f(x) = +\infty$

$x = 0$

$\lim\limits_{x \to +\infty} f(x) = +\infty$

Нема К.А.

5) $T_{min}\left(e^2-1, \frac{e^2}{4}\right)$

6) $$y'' = \left(\frac{\ln(x+1)-2}{\ln^3(x+1)}\right)' = \frac{\frac{1}{x+1}\cdot\ln^3(x+1) - (\ln(x+1)-2)\cdot 3\ln^2(x+1)\cdot\frac{1}{x+1}}{\ln^6(x+1)}$$

$$= \frac{\ln^3(x+1) - 3\ln^3(x+1) + 6\ln^2(x+1)}{(x+1)\cdot\ln^6(x+1)} = \frac{-2\ln^3(x+1) + 6\ln^2(x+1)}{(x+1)\ln^6(x+1)}$$

$$= \frac{2\ln^2(x+1)(3-\ln(x+1))}{(x+1)\ln^6(x+1)} = \frac{2(3-\ln(x+1))}{(x+1)\cdot\ln^4(x+1)}$$

$$y''=0 \iff 3-\ln(x+1)=0$$
$$3=\ln(x+1)$$
$$x+1=e^3$$
$$x=e^3-1$$

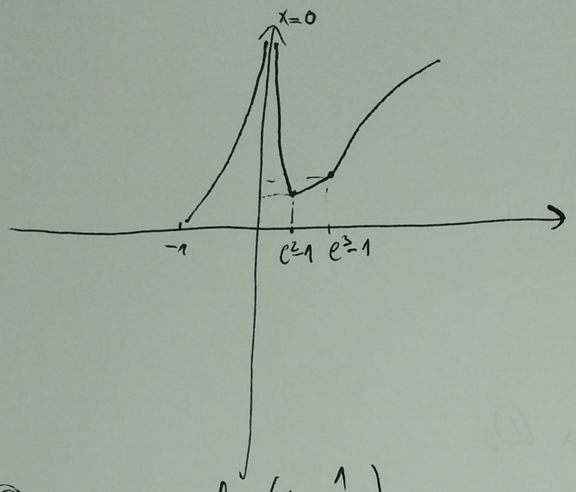
$y''>0 \iff 3-\ln(x+1)>0$ јер $x+1>0$ и $\ln^4(x+1)>0$
$$\ln(x+1)<3$$
$$x+1<e^3$$
$$x<e^3-1$$

| | $-1$ | $0$ | | $e^3-1$ | $+\infty$ |
|---|---|---|---|---|---|
| $y''$ | $+$ | | $+$ | $0$ | $-$ |
| $y$ | $\cup$ | | $\cup$ | превојна | $\cap$ |

$$P(e^3-1, f(e^3-1))$$

$$f(e^3-1) = \frac{e^3-\cancel{1}+\cancel{1}}{\ln^2(e^3-\cancel{1}+1)} = \frac{e^3}{3^2} = \frac{e^3}{9}$$

$$P\left(e^3-1, \frac{e^3}{9}\right)$$




⑧ $y = \operatorname{arctg}\left(1+\frac{1}{x}\right)$

1) $D_f = \mathbb{R} \setminus \{0\}$

2) $f(-x) = \operatorname{arctg}\left(1+\frac{1}{-x}\right) = \operatorname{arctg}\left(1-\frac{1}{x}\right) \neq f(x)$, $\neq -f(x)$

Ни чётна ни нечётна

3) Нуле и знак

$f(x)=0 \iff \operatorname{arctg}\left(1+\frac{1}{x}\right) = 0$

$1+\frac{1}{x} = 0$

$\frac{1}{x} = -1$

$x = -1$

$N(-1, 0)$

$f(x)>0 \iff \operatorname{arctg}\left(1+\frac{1}{x}\right) > 0$

$1+\frac{1}{x} > 0$

$\frac{1}{x} > -1$

$\frac{x+1}{x} > 0$

$\boxed{x \in (-\infty, -1) \cup (0, +\infty)}$

| | $-\infty$ | $-1$ | | $0$ | $+\infty$ |
|---|---|---|---|---|---|
| $x+1$ | $-$ | $0$ | $+$ | $+$ | $+$ |
| $x$ | $-$ | | $-$ $-$ | $0$ | $+$ |
| $\frac{x+1}{x}$ | $+$ | $0$ | $-$ | // | $+$ |

$f(x)<0 \iff \operatorname{arctg}\left(1+\frac{1}{x}\right) < 0$

$x \in (-1, 0)$

4) Асимптоте

B.A Няма

$\lim_{x \to 0+} f(x) = \frac{\pi}{2}$

$\lim_{x \to 0-} f(x) = -\frac{\pi}{2}$

$y = \frac{\pi}{4}$ Х.А $(x \to \pm\infty)$

Няма К.А

5) $f'(x) < 0 \ \forall x \in D_f \Rightarrow f \downarrow$ на $D_f$

6) $f''(x) = \left(\frac{-1}{2x^2+2x+1}\right)' = -1 \cdot \frac{(-1)}{(2x^2+2x+1)^2} \cdot (4x+2) = \frac{2(2x+1)}{(2x^2+2x+1)^2}$

$f''(x) = 0 \Leftrightarrow 2x+1 = 0$

$x = -\frac{1}{2}$

$f''(x) > 0 \Leftrightarrow \frac{2(2x+1)}{(2x^2+2x+1)^2} > 0 \Leftrightarrow 2x+1 > 0$

$(2x^2+2x+1)^2 > 0$ $\qquad x > -\frac{1}{2}$

$f''(x) < 0 \Leftrightarrow x < -\frac{1}{2}$

| $x$ | | $-\frac{1}{2}$ | | $0$ | $+\infty$ |
|---|---|---|---|---|---|
| $f''(x)$ | $-$ | $0$ | $+$ | $+$ | $+$ |
| $f(x)$ | $\frown$ | пре-вој | $\smile$ | | $\smile$ |

$P\left(-\frac{1}{2}, -\frac{\pi}{4}\right)$