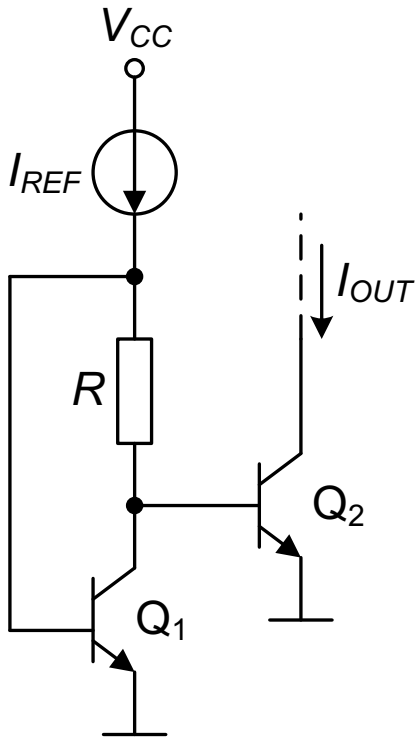
VJEŽBE 8

OSNOVE ELEKTRONIKE, ETR, IV SEMESTAR

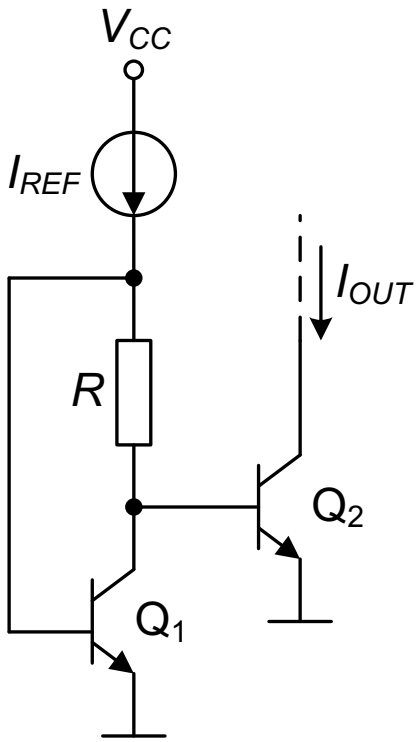
DOC. DR MILENA ERCEG

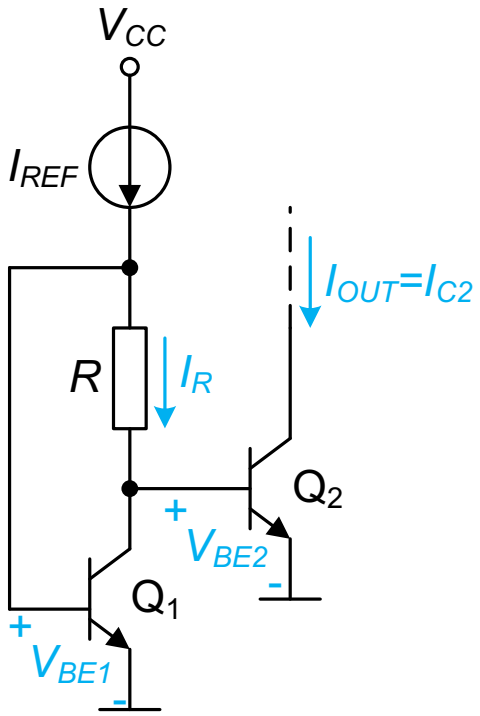
ZADATAK 1

Za kolo prikazano na slici izračunati struju I_{OUT}. Bipolarni tranzistori su identičnih karakteristika, $V_{Ai} = V_A \longrightarrow \infty$, $V_{Ti} = V_T = 26$ mV, $I_{Si} = I_S$, $\beta_i = \beta \rightarrow \infty$, $i = \overline{1,2}$, struja strujnog izvora $I_{REF} = 10$ μA i otpornost $R = 12$ kΩ. Ukoliko strujno pojačanje β i *Early*-jev napon imaju konačnu vrijednost, odrediti izlaznu otpornost kola. Bipolarni tranzistor Q_2 je u direktnom aktivnom režimu.

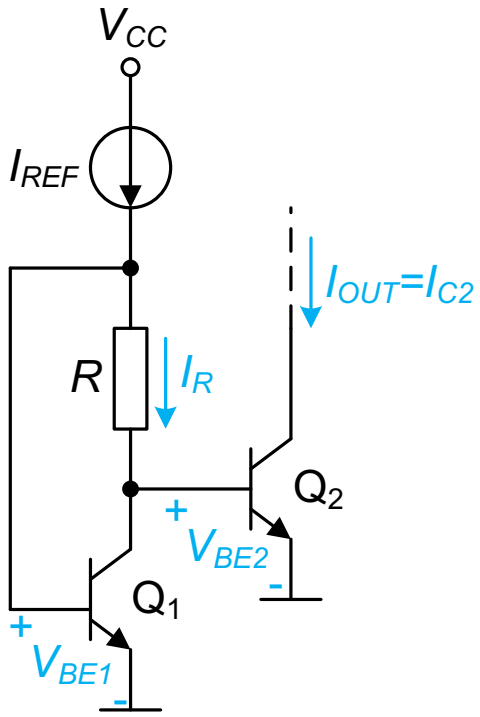


ZADATAK 1 – DC ANALIZA





ZADATAK 1 – DC ANALIZA



$$I_{OUT} = I_{C2} = I_{S2}e^{\frac{V_{BE2}}{V_T}}$$

$$\beta \to \infty \Longrightarrow I_{C1} \approx I_{REF}, I_{B1} \to 0$$

$$V_{BE2} = V_{BE1} - RI_R \approx V_{BE1} - RI_{REF}$$

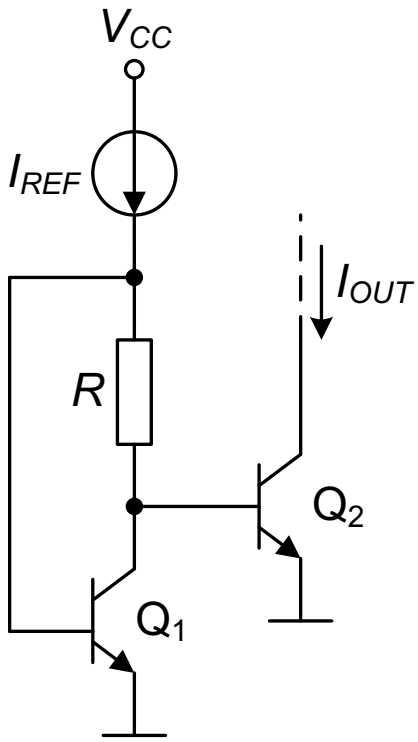
$$I_{OUT} = I_{S2}e^{\frac{V_{BE1}-RI_{REF}}{V_T}} = I_{S1}e^{\frac{V_{BE1}}{V_T}}e^{\frac{-RI_{REF}}{V_T}} = I_{C1}e^{\frac{-RI_{REF}}{V_T}}$$

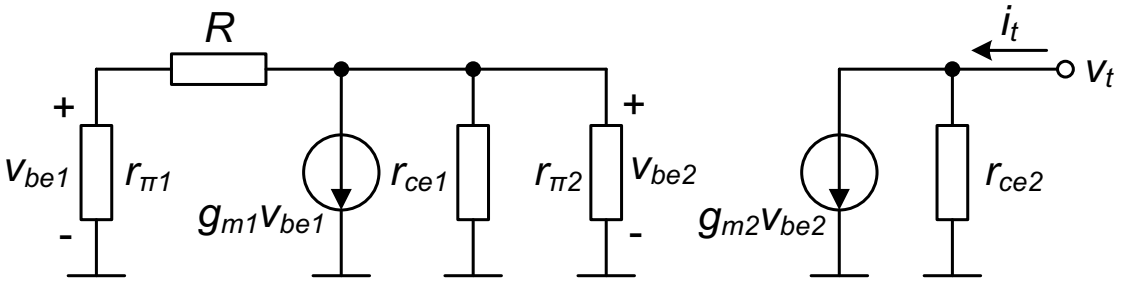
$$I_{OUT} \approx I_{REF}e^{\frac{-RI_{REF}}{V_T}} = 98.98\ \text{nA}$$

$$V_{CE1} = V_{BE2}$$

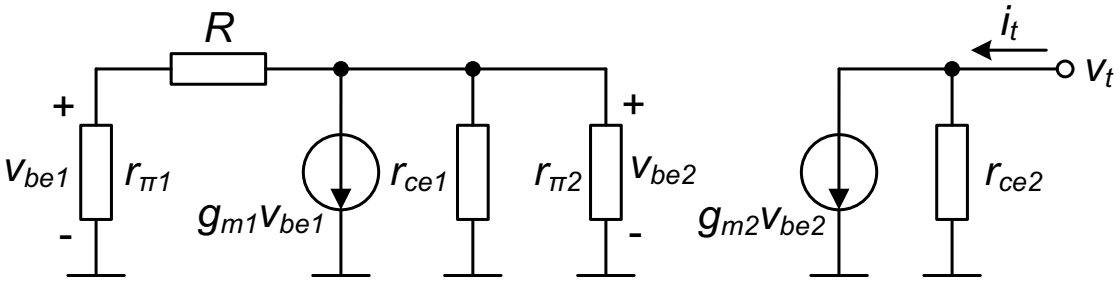
Slijedi da je i BJT Q_1 u direktnom aktivnom režimu.

ZADATAK 1 – AC ANALIZA





ZADATAK 1 – AC ANALIZA



$$\frac{v_{be1}}{r_{\pi 1}} + g_{m1}v_{be1} + \frac{v_{be2}}{r_{ce1}} + \frac{v_{be2}}{r_{\pi 2}} = 0$$

$$v_{be1} = \frac{r_{\pi 1}}{r_{\pi 1} + R} v_{be2}$$

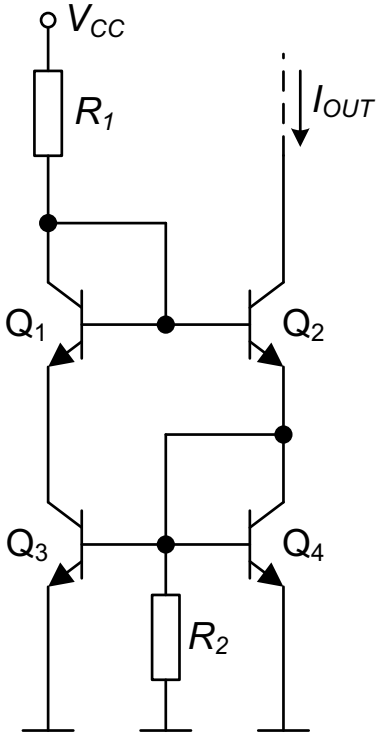
$$\left[\frac{r_{\pi 1}}{r_{\pi 1} + R}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right) + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi 2}}\right] v_{be2} = 0$$

$$\Rightarrow v_{be1} = 0, v_{be2} = 0$$

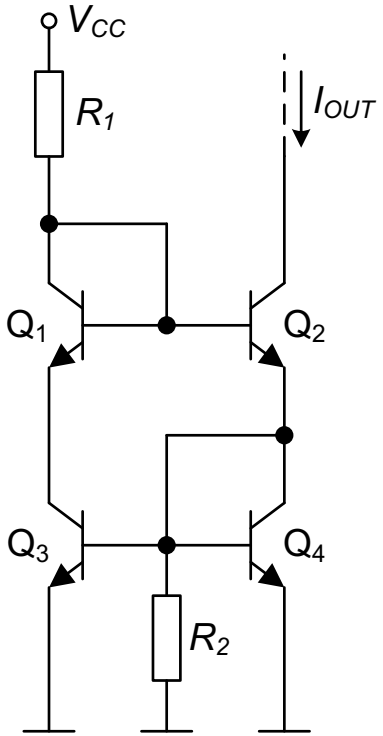
$$R_{out} = \frac{v_t}{i_t} = r_{ce2}$$

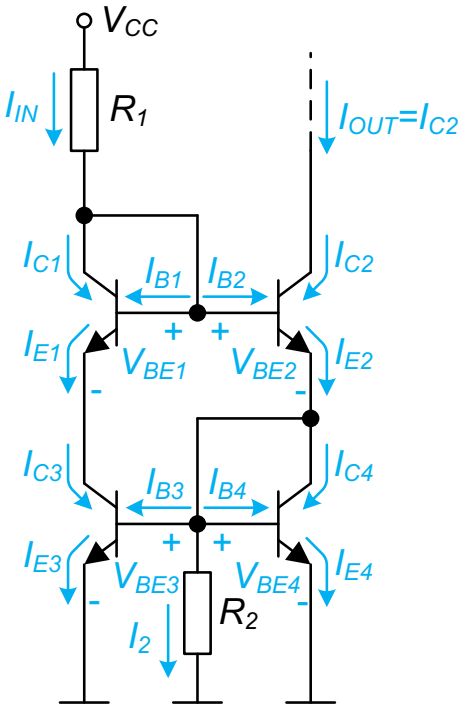
ZADATAK 2

Za kolo prikazano na slici izračunati odnos otpornosti (R_1 / R_2) tako da izlazna struja I_{OUT} ne zavisi od napona baza-emitor. Bipolarni tranzistori su identičnih karakteristika, $V_{Ai} = V_A \longrightarrow \infty$, $V_{Ti} = V_T$, $I_{Si} = I_S$, $\beta_i = \beta$, $i = \overline{1,4}$. Bipolarni tranzistor Q_2 je u direktnom aktivnom režimu.

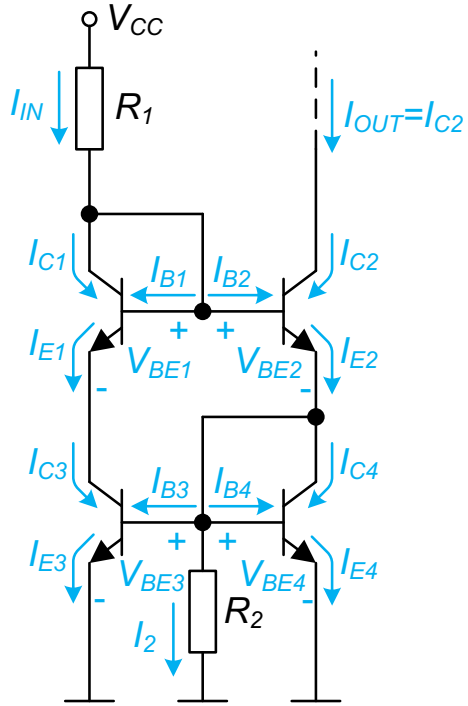


ZADATAK 2


V_{CC}
R_1
I_{OUT}
Q_1
Q_2
Q_3
Q_4
R_2


V_{CC}
I_{IN}
R_1
$I_{OUT}=I_{C2}$
I_{C1}
I_{B1}
I_{B2}
I_{C2}
I_{E1}
V_{BE1}
V_{BE2}
I_{E2}
I_{C3}
I_{B3}
I_{B4}
I_{C4}
I_{E3}
V_{BE3}
V_{BE4}
I_{E4}
R_2
I_2

ZADATAK 2



$$V_{CE1} = V_{BE1}$$

$$V_{CE3} = -V_{BE1} + V_{BE2} + V_{BE4}$$

$$V_{CE4} = V_{BE4}$$

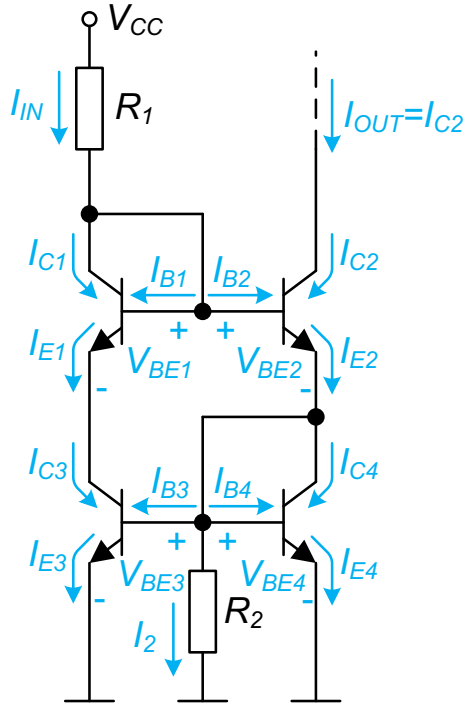
Slijedi da su i bipolarni tranzistori Q_1, Q_3 i Q_4 također u direktnom aktivnom režimu.

$$I_{OUT} = I_{C2} = \beta I_{B2}$$

$$I_{B2} = I_{IN} - I_{C1} - I_{B1} = I_{IN} - I_{E1} = I_{IN} - I_{C3}$$

$$I_{IN} = \frac{V_{CC} - V_{BE2} - V_{BE4}}{R_1}$$

ZADATAK 2



$$V_{BE3} = V_{BE4} \Longrightarrow I_{C3} = I_{C4}$$

$$I_{E1} = I_{C3} = I_{C4}$$

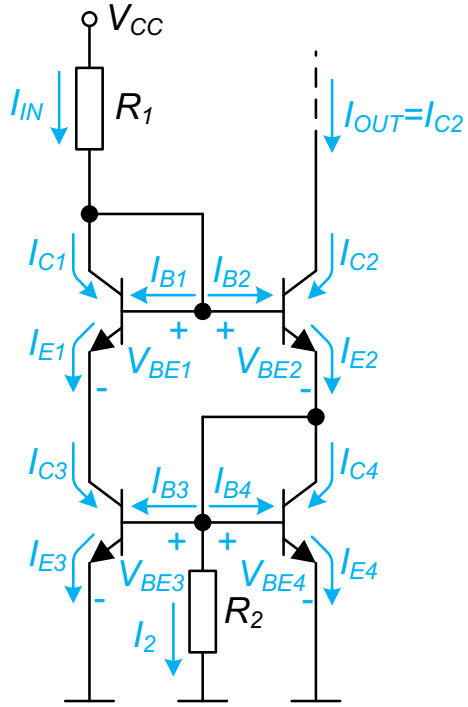
$$I_{C4} = I_{E2} - I_{B3} - I_{B4} - I_2 = I_{E2} - 2I_{B4} - \frac{V_{BE4}}{R_2}$$

$$I_{C4} = (\beta + 1)I_{B2} - 2I_{B4} - \frac{V_{BE4}}{R_2}$$

$$(\beta + 2)I_{B4} = (\beta + 1)I_{B2} - \frac{V_{BE4}}{R_2}$$

$$I_{C4} = \frac{\beta}{\beta + 2}\left[(\beta + 1)I_{B2} - \frac{V_{BE4}}{R_2}\right]$$

ZADATAK 2



$$I_{B2} = I_{IN} - I_{C1} - I_{B1} = I_{IN} - I_{E1} = I_{IN} - I_{C3}$$

$$I_{C3} = I_{C4} = \frac{\beta}{\beta + 2}\left[(\beta + 1)I_{B2} - \frac{V_{BE4}}{R_2}\right]$$

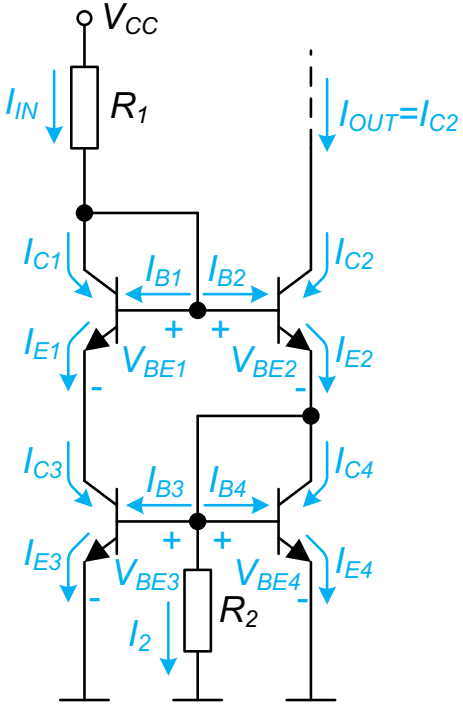
$$I_{B2} = \frac{V_{CC} - V_{BE2} - V_{BE4}}{R_1} - \frac{\beta}{\beta + 2}\left[(\beta + 1)I_{B2} - \frac{V_{BE4}}{R_2}\right]$$

$$I_{B2} = \frac{\beta + 2}{\beta^2 + 2\beta + 2}\left(\frac{V_{CC} - V_{BE2} - V_{BE4}}{R_1} + \frac{\beta}{\beta + 2}\frac{V_{BE4}}{R_2}\right)$$

$$I_{OUT} = \beta I_{B2}$$

$$I_{OUT} = \beta\frac{\beta + 2}{\beta^2 + 2\beta + 2}\left(\frac{V_{CC} - V_{BE2} - V_{BE4}}{R_1} + \frac{\beta}{\beta + 2}\frac{V_{BE4}}{R_2}\right)$$

ZADATAK 2



$$I_{OUT} = \beta \frac{\beta + 2}{\beta^2 + 2\beta + 2}\left(\frac{V_{CC} - V_{BE2} - V_{BE4}}{R_1} + \frac{\beta}{\beta + 2}\frac{V_{BE4}}{R_2}\right)$$

$$\frac{-V_{BE2} - V_{BE4}}{R_1} + \frac{\beta}{\beta + 2}\frac{V_{BE4}}{R_2} = 0$$

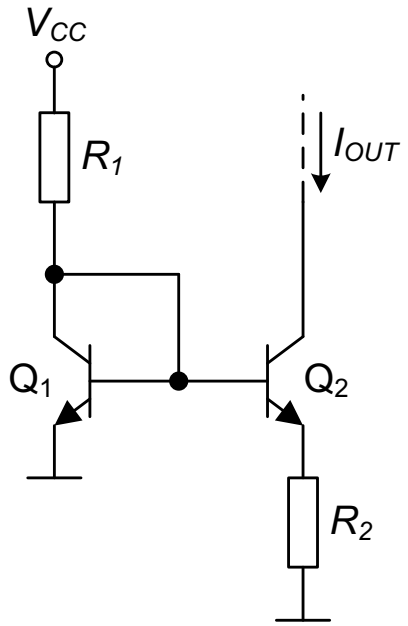
$$\frac{R_1}{R_2} = \frac{\beta + 2}{\beta}\frac{V_{BE2} + V_{BE4}}{V_{BE4}} \approx 2$$

Ukoliko se odnos otpornosti (R_1 / R_2) izabere prema prethodnoj relaciji, izlazna struja je data sljedećim izrazom:

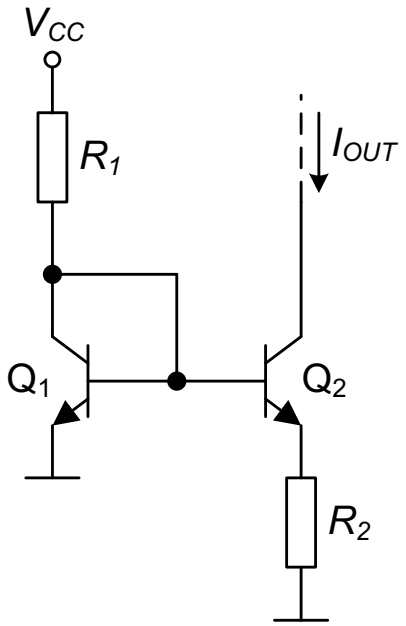
$$I_{OUT} = \beta \frac{\beta + 2}{\beta^2 + 2\beta + 2}\frac{V_{CC}}{R_1} \approx \frac{V_{CC}}{R_1}$$

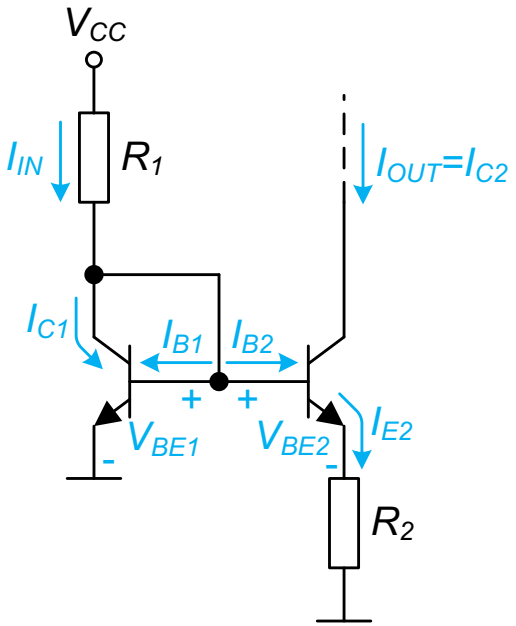
ZADATAK 3

Za kolo prikazano na slici izračunati otpornost R_2 tako da izlazna struja iznosi $I_{OUT} = 10$ μA. Poznat je napon napajanja kola $V_{CC} = 30$ V i otpornost $R_1 = 29.3$ kΩ. Bipolarni tranzistori su identičnih karakteristika, $V_{Ai} = V_A \longrightarrow \infty$, $V_{Ti} = V_T = 26$ mV, $I_{Si} = I_S$, $\beta_i = \beta \longrightarrow \infty$, $i = \overline{1,2}$. Bipolarni tranzistor Q_2 je u direktnom aktivnom režimu. Ukoliko *Early*-jev napon V_A i strujno pojačanje β imaju konačnu vrijednost, odrediti izlaznu otpornost kola.

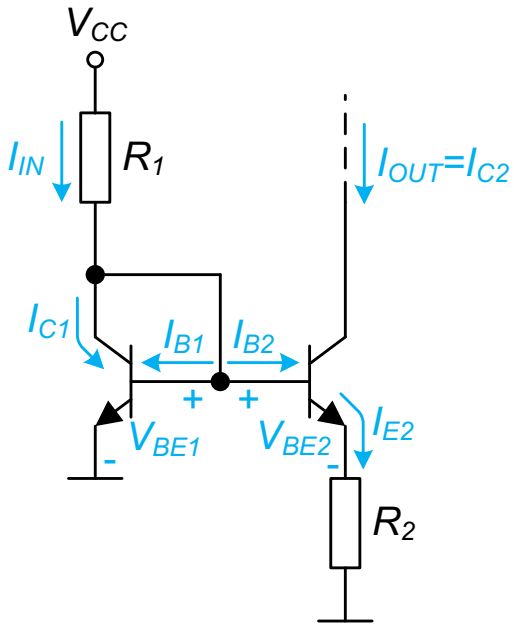


ZADATAK 3 – DC ANALIZA





ZADATAK 3 – DC ANALIZA



$$V_{CE1} = V_{BE1}$$

Slijedi da je i BJT Q_1 u direktnom aktivnom režimu.

$$I_{OUT} = I_{S2}e^{\frac{V_{BE2}}{V_T}} = I_{S2}e^{\frac{V_{BE1}-R_2I_{E2}}{V_T}}$$

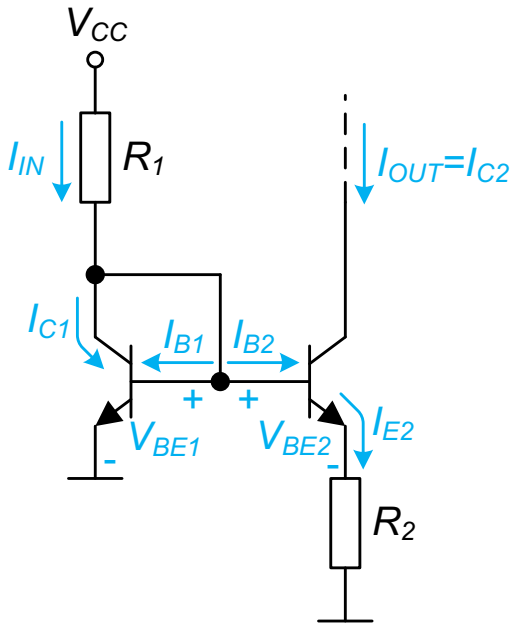
$$\beta \to \infty \Longrightarrow I_{C2} \approx I_{E2},\ I_{C1} \approx I_{E1},\ I_{B1}, I_{B2} \to 0$$

$$I_{C1} \approx I_{IN} = \frac{V_{CC} - V_{BE1}}{R_1} = 1\ \text{mA}$$

$$I_{OUT} = I_{S2}e^{\frac{V_{BE1}-R_2I_{OUT}}{V_T}} = I_{S1}e^{\frac{V_{BE1}}{V_T}}e^{\frac{-R_2I_{OUT}}{V_T}} = I_{C1}e^{\frac{-R_2I_{OUT}}{V_T}}$$

$$I_{OUT} = \frac{V_{CC} - V_{BE1}}{R_1}e^{\frac{-R_2I_{OUT}}{V_T}}$$

ZADATAK 3 – DC ANALIZA

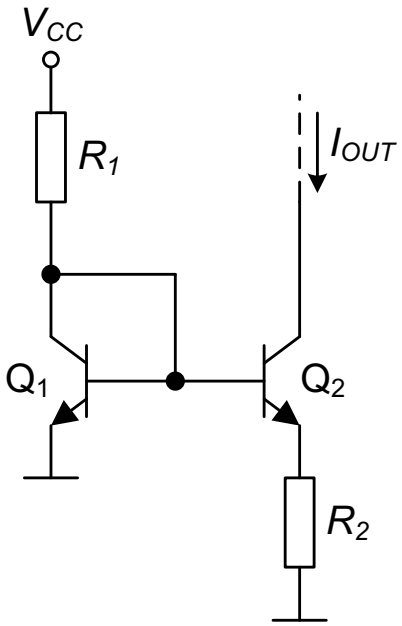


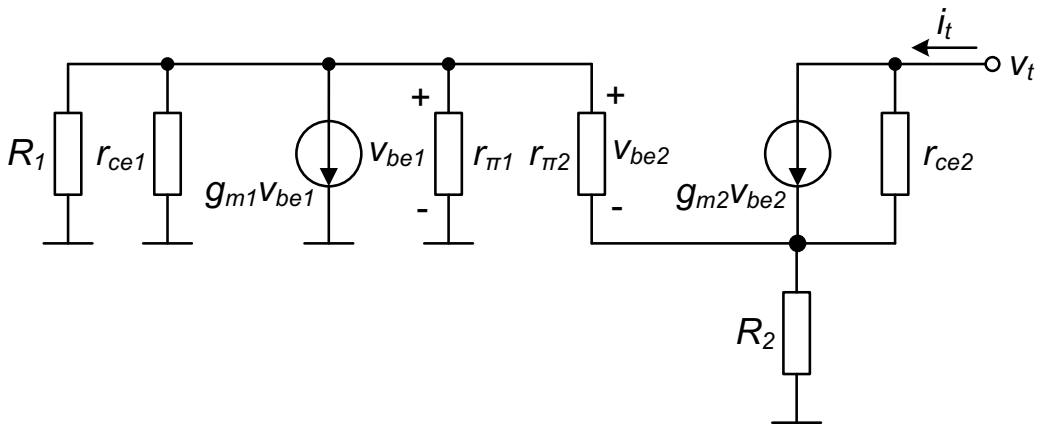
$$I_{OUT} = \frac{V_{CC} - V_{BE1}}{R_1} e^{\frac{-R_2 I_{OUT}}{V_T}}$$

$$R_2 = \frac{V_T}{I_{OUT}} \ln\left(\frac{V_{CC} - V_{BE1}}{R_1 I_{OUT}}\right)$$

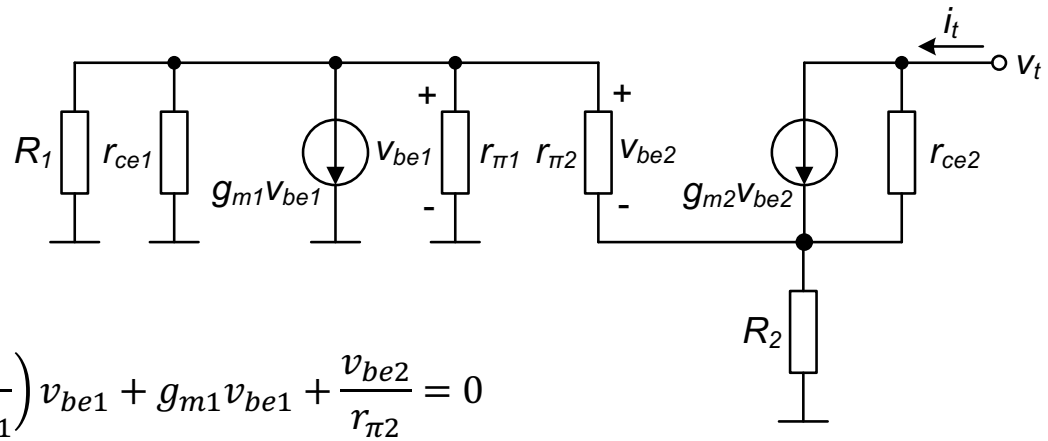
Ukoliko je napon V_{BE1} = 0.7 V, otpornost R_2 će biti približno jednaka 11.97 kΩ. Treba napomenuti da varijacija napona V_{BE} za $\pm$10 % ne utiče značajno na rezultat jer je napon napajanja 30 V.

ZADATAK 3 – AC ANALIZA





ZADATAK 3 – AC ANALIZA



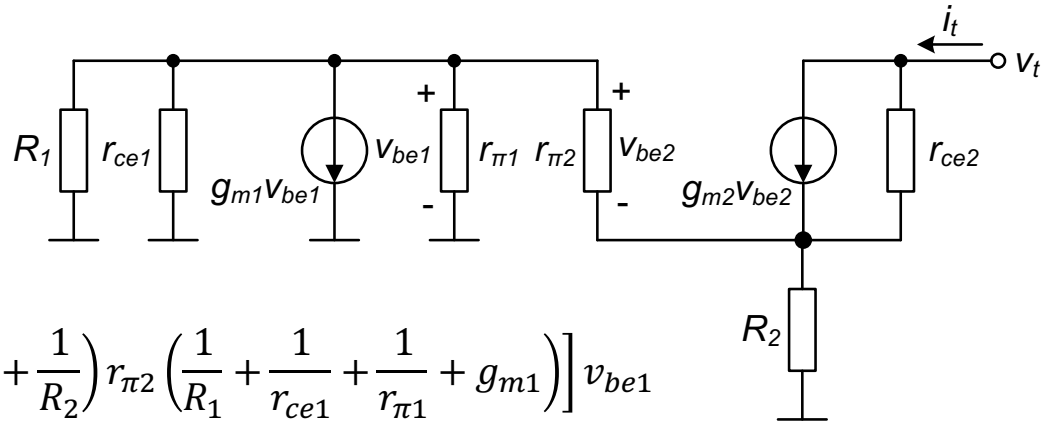
$$\left(\frac{1}{R_1}+\frac{1}{r_{ce1}}+\frac{1}{r_{\pi1}}\right)v_{be1}+g_{m1}v_{be1}+\frac{v_{be2}}{r_{\pi2}}=0$$

$$v_{be2}=-r_{\pi2}\left(\frac{1}{R_1}+\frac{1}{r_{ce1}}+\frac{1}{r_{\pi1}}+g_{m1}\right)v_{be1}$$

$$i_t+\frac{v_{be2}}{r_{\pi2}}=\frac{v_{be1}-v_{be2}}{R_2}\Rightarrow i_t=\frac{v_{be1}}{R_2}-\left(\frac{1}{r_{\pi2}}+\frac{1}{R_2}\right)v_{be2}$$

$$i_t=\left[\frac{1}{R_2}+\left(\frac{1}{r_{\pi2}}+\frac{1}{R_2}\right)r_{\pi2}\left(\frac{1}{R_1}+\frac{1}{r_{ce1}}+\frac{1}{r_{\pi1}}+g_{m1}\right)\right]v_{be1}$$

ZADATAK 3 – AC ANALIZA



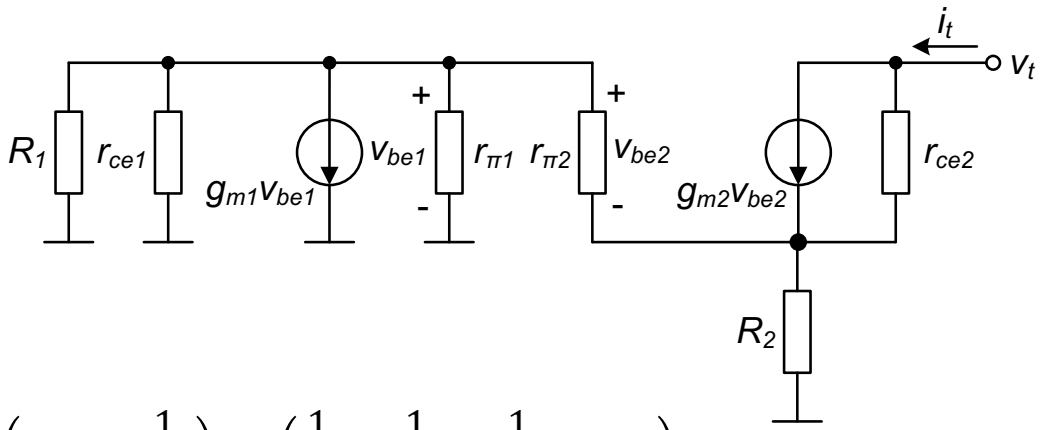
$$i_t = \left[\frac{1}{R_2} + \left(\frac{1}{r_{\pi2}} + \frac{1}{R_2}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)\right] v_{be1}$$

$$v_{be1} = \frac{i_t}{\frac{1}{R_2} + \left(\frac{1}{r_{\pi2}} + \frac{1}{R_2}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)}$$

$$i_t = g_{m2} v_{be2} + \frac{v_t - v_{be1} + v_{be2}}{r_{ce2}} = \frac{v_t - v_{be1}}{r_{ce2}} + \left(g_{m2} + \frac{1}{r_{ce2}}\right) v_{be2}$$

$$i_t = \frac{v_t - v_{be1}}{r_{ce2}} - \left(g_{m2} + \frac{1}{r_{ce2}}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right) v_{be1}$$

ZADATAK 3 – AC ANALIZA

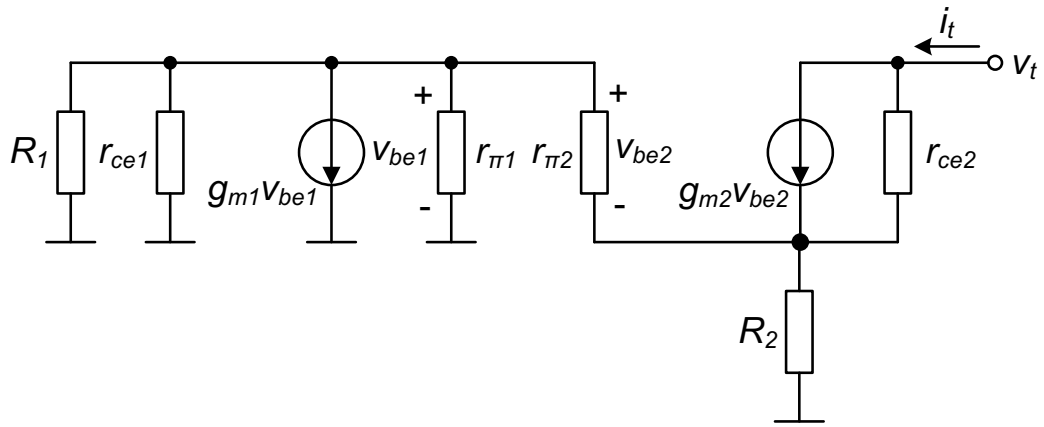


$$i_t = \frac{v_t - v_{be1}}{r_{ce2}} - \left(g_{m2} + \frac{1}{r_{ce2}}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right) v_{be1}$$

$$i_t = \frac{v_t}{r_{ce2}} - \left[\frac{1}{r_{ce2}} + \left(g_{m2} + \frac{1}{r_{ce2}}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)\right] v_{be1}$$

$$i_t = \frac{v_t}{r_{ce2}} - \frac{\frac{1}{r_{ce2}} + \left(g_{m2} + \frac{1}{r_{ce2}}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)}{\frac{1}{R_2} + \left(\frac{1}{r_{\pi2}} + \frac{1}{R_2}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)} i_t$$

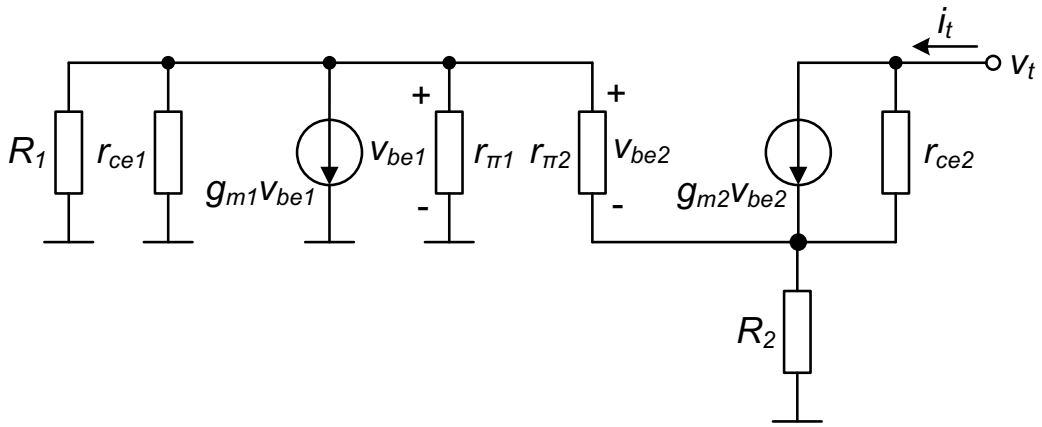
ZADATAK 3 – AC ANALIZA



$$i_t = \frac{v_t}{r_{ce2}} - \frac{\frac{1}{r_{ce2}} + \left(g_{m2} + \frac{1}{r_{ce2}}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)}{\frac{1}{R_2} + \left(\frac{1}{r_{\pi2}} + \frac{1}{R_2}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)} i_t$$

$$R_{out} = \frac{v_t}{i_t} = r_{ce2} \left[1 + \frac{\frac{1}{r_{ce2}} + \left(g_{m2} + \frac{1}{r_{ce2}}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)}{\frac{1}{R_2} + \left(\frac{1}{r_{\pi2}} + \frac{1}{R_2}\right) r_{\pi2} \left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)}\right]$$

ZADATAK 3 – AC ANALIZA



$$R_{out} = \frac{v_t}{i_t} = r_{ce2}\left[1 + \frac{\frac{1}{r_{ce2}} + \left(g_{m2} + \frac{1}{r_{ce2}}\right) r_{\pi2}\left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)}{\frac{1}{R_2} + \left(\frac{1}{r_{\pi2}} + \frac{1}{R_2}\right) r_{\pi2}\left(\frac{1}{R_1} + \frac{1}{r_{ce1}} + \frac{1}{r_{\pi1}} + g_{m1}\right)}\right]$$

$g_{m1}r_{\pi1} \gg 1, g_{m1}r_{ce1} \gg 1, g_{m2}r_{\pi2} \gg 1, g_{m2}r_{ce2} \gg 1$:

$$R_{out} \approx r_{ce2}\left[1 + \frac{\frac{1}{r_{ce2}} + g_{m2}r_{\pi2}\left(\frac{1}{R_1} + g_{m1}\right)}{\frac{1}{R_2} + \left(\frac{1}{r_{\pi2}} + \frac{1}{R_2}\right) r_{\pi2}\left(\frac{1}{R_1} + g_{m1}\right)}\right]$$

ZADATAK 3 – AC ANALIZA

$$R_{out} \approx r_{ce2}\left[1+\frac{\frac{1}{r_{ce2}}+g_{m2}r_{\pi2}\left(\frac{1}{R_1}+g_{m1}\right)}{\frac{1}{R_2}+\left(\frac{1}{r_{\pi2}}+\frac{1}{R_2}\right)r_{\pi2}\left(\frac{1}{R_1}+g_{m1}\right)}\right]$$

Kako je izlazna struja I_{OUT} 100 puta manja od ulazne struje I_{IN}, transkonduktansa BJT-a Q_1 je 100 puta veća od transkonduktanse BJT-a Q_2, dok je izlazna otpornost BJT-a Q_1 100 puta manja od izlazne otpornosti BJT-a Q_2. Slijedi da se izraz za izlaznu otpornost dalje može aproksimirati:

$$R_{out} \approx r_{ce2}\left[1+\frac{\frac{1}{r_{ce2}}+g_{m2}r_{\pi2}\left(\frac{1}{R_1}+100g_{m2}\right)}{\frac{1}{R_2}+\left(\frac{1}{r_{\pi2}}+\frac{1}{R_2}\right)r_{\pi2}\left(\frac{1}{R_1}+100g_{m2}\right)}\right] \approx r_{ce2}\left[1+\frac{g_{m2}}{\frac{1}{R_2r_{\pi2}\left(\frac{1}{R_1}+100g_{m2}\right)}+\frac{1}{r_{\pi2}}+\frac{1}{R_2}}\right]$$

$$\approx r_{ce2}\left(1+\frac{g_{m2}}{\frac{1}{r_{\pi2}}+\frac{1}{R_2}}\right)$$

ZADATAK 3 – AC ANALIZA

$$R_{out} \approx r_{ce2}\left[1+\frac{\frac{1}{r_{ce2}}+g_{m2}r_{\pi2}\left(\frac{1}{R_1}+100g_{m2}\right)}{\frac{1}{R_2}+\left(\frac{1}{r_{\pi2}}+\frac{1}{R_2}\right)r_{\pi2}\left(\frac{1}{R_1}+100g_{m2}\right)}\right] \approx r_{ce2}\left[1+\frac{g_{m2}}{\frac{1}{R_2r_{\pi2}\left(\frac{1}{R_1}+100g_{m2}\right)}+\frac{1}{r_{\pi2}}+\frac{1}{R_2}}\right]$$

$$\approx r_{ce2}\left(1+\frac{g_{m2}}{\frac{1}{r_{\pi2}}+\frac{1}{R_2}}\right)$$

Kako je kolektorska struja BJT-a Q_2: I_{C2} = 10 μA, transkonduktansa iznosi $g_{m2} = (I_{C2} / V_T) =$ 0.385 mS. Za strujno pojačanje β reda stotinu, otpornost $r_{\pi2}$ je reda 2-3 stotine kΩ, što je značajno veće od R_2. Konačan izraz za izlaznu otpornost bi bio:

$$R_{out} \approx r_{ce2}(1+g_{m2}R_2) = 5.61r_{ce2}$$