Račun sa približnim vrijednostima

- greška i relativna greška
- maksimalna apsolutna greška i maksimalna relativna greška

x - tačna vrijednost, $\bar{x}$ - približna vrijednost,

$e_x = \bar{x} - x$ greška, $r_x = \frac{e_x}{x} = \frac{\bar{x} - x}{x} \approx \frac{e_x}{\bar{x}}$ relativna greška,

$e_{x\pm y} = e_x \pm e_y$, $r_{x+y} = \frac{x}{x+y} r_x + \frac{y}{x+y} r_y$, $r_{xy} = r_x + r_y$ $r_{x/y} = r_x - r_y$

Δ_x - maksimalna apsolutna greška, $\varepsilon_x = \frac{\Delta_x}{|x|} \approx \frac{\Delta_x}{|\bar{x}|}$, maksimalna relativna greška

$\Delta_{x\pm y} = \Delta_x + \Delta_y$, $\varepsilon_{xy} = \varepsilon_x + \varepsilon_y$, $\varepsilon_{x/y} = \varepsilon_x + \varepsilon_y$

1. Kolika je ~~apsolutna~~ a kolika relativna greška učinjena zaokruživanjem broja x=1.25138 na a) 2, b) 3 i c) 4 decimale?

a) $\bar{x} = 1.25$ $e_x = \bar{x} - x = -0.00138$ $r_x = -0.00110278$

b) $\bar{x} = 1.251$ $e_x = -0.00038$ $r_x = -0.0003036648$

c) $\bar{x} = 1.2514$ $e_x = 0.00002$ $r_x = 0.00001598236$

2. Računa se funkcija $f = xy + 2z$. Poznate su približne vrijednosti promjenjivih x, y i z i one iznose: 1, 3 i –1 respektivno. Takođe su poznate i maksimalne relativne greške za svaku promjenjivu i one iznose: 10^{-3}, $3\ 10^{-3}$ i 10^{-4}. Sračunati: $\bar{f}$, Δ_f i ε_f

$\bar{f} = \bar{x}\bar{y} + 2\bar{z} = 1$, $\varepsilon_{xy} = \varepsilon_x + \varepsilon_y$, $\varepsilon_{2z} = \varepsilon_z$,

$\Delta_f = \Delta_{xy} + \Delta_{2z} = |\bar{x}\bar{y}|\varepsilon_{xy} + |2\bar{z}|\varepsilon_{2z} = |\bar{x}\bar{y}|(\varepsilon_x + \varepsilon_y) + |2\bar{z}|\varepsilon_z = 4.2 \cdot 10^{-3}$, $\varepsilon_f = \Delta_f / \bar{f} = 4.2 \cdot 10^{-3}$

3. Brojevi x i y su poznati sa 5 tačnih decimala. $\bar{x} = 1.23456$ i $\bar{y} = 1.23465$. Znajući da su nastali zaokruživanjem, sračunati maksimalnu apsolutnu i maksimalnu relativnu grešku nastalu pri određivanju ovih veličina, a nakon toga odrediti maksimalnu apsolutnu i maksimalnu relativnu grešku njihovog zbira i razlike.

mrx = 4.0500 e -6
mry = 4.0497 e -6

mrg zbira	4.0499e-006	mag zbira	0.000001	x+y = 2.46921
mrg razlike	0.111	mag razlike	0.000001	x-y = 0.0009

4. Vrši se računanje sume $S = \sum_{n=1}^{1000000} x_n$ gdje su x_n brojevi zapisani sa maksimalnom apsolutnom greškom $\Delta_{x_n} = 10^{-7}$. Dobijen je rezultat $S = 2.1325623$. Sračunati maksimalnu apsolutnu grešku i maksimalnu relativnu grešku sume S. Prokomentarisati koliko tačnih cifara možemo očekivati u rezultatu.

mag 0.1
rg 0.0469

5. $F = \frac{x + y^2}{xy - \frac{x}{y} + \frac{y}{2}}$ $\Delta_x = \Delta_y = 0.02$; $\bar{x} = 1$, $\bar{y} = 2$

Matematika a počítačové

1. $x = 1.25138$

a) $\bar{x} = 1.25$

$e_x = \bar{x} - x = 1.25 - 1.25138 = -0.00138$

$\varepsilon_x = \frac{e_x}{x} = -0.001102783$

b) $\bar{x} = 1.251$

$e_x = \bar{x} - x = -0.00038$

$\varepsilon_x = \frac{e_x}{x} = -\frac{0.00038}{1.25138} = -0.000303664$8

c) $\bar{x} = 1.254$

$e_x = \bar{x} - x = 1.254 - 1.25138 = 0.00002$

$\varepsilon_x = \frac{e_x}{x} = 0.000015982$

2. $f = xy + 2z$

$\bar{x} = 1 \quad \varepsilon_x = 10^{-3}$

$\bar{y} = 3 \quad \varepsilon_y = 3 \cdot 10^{-3}$

$\bar{z} = -1 \quad \varepsilon_z = 10^{-4}$

$\bar{f} = ? \quad \Delta f = ? \quad \varepsilon_f = ?$

$\bar{f} = \bar{x}\bar{y} + 2\bar{z} = 1 \cdot 3 - 2 = 1$

$\varepsilon_f = \varepsilon_{xy} + \varepsilon_{2z}$

$\varepsilon_{2z} = \varepsilon_z$

$$\varepsilon_{xy} = \frac{\Delta_{xy}}{|\bar{x} \cdot \bar{y}|} = \varepsilon_x + \varepsilon_y$$

$$\varepsilon_f = \varepsilon_x + \varepsilon_y + \varepsilon_z = 10^{-3} + 3 \cdot 10^{-3} + 10^{-4} = 4 \cdot 10^{-3} + 0.1 \cdot 10^{-3} \cdot 2 =$$
$$= 4.2 \cdot 10^{-3}$$

$$\Delta_f = \Delta_{xy} + \Delta_{2z} = \varepsilon_{xy} \cdot |\bar{x}\bar{y}| + \varepsilon_{2z} \cdot |2\bar{z}| =$$
$$= (\varepsilon_x + \varepsilon_y)|\bar{x} \cdot \bar{y}| + |2\bar{z}| \cdot \varepsilon_z = 12.2 \cdot 10^{-3}$$

3. $\bar{x} = 1.23456$
$\bar{y} = 1.23465$

tada će greška biti maksimalna

$$\Delta_x = \bar{x} - x = 1.23456 - 1.234555 = 5 \cdot 10^{-6}$$
$$\Delta_y = \bar{y} - y = 1.23465 - 1.234645 = 5 \cdot 10^{-6}$$

$$\varepsilon_x = \frac{\Delta x}{|\bar{x}|} = \frac{5 \cdot 10^{-6}}{1.23456} = 4.05 \cdot 10^{-6}$$

$$\varepsilon_y = \frac{\Delta y}{|\bar{y}|} = \frac{5 \cdot 10^{-6}}{1.23465} = 4.05 \cdot 10^{-6}$$

$$\Delta_{x+y} = \Delta_x + \Delta_y = 10^{-5}$$

$$\Delta_{x-y} = \Delta_x + \Delta_y = 10^{-5}$$

$$\varepsilon_{x+y} = \frac{\Delta_{x+y}}{|\bar{x}+\bar{y}|} = \frac{10^{-5}}{|1.23456 + 1.23465|} \quad 4.05 \cdot 10^{-6}$$

$$\varepsilon_{x-y} = \frac{\Delta_{x-y}}{|\bar{x}-\bar{y}|} = 0.111$$

4. $S = \sum_{n=1} x_n$

$\Delta x_n = 10^{-7}$

$S = 2.1325623$

$\Delta_S = \Delta x_1 + \Delta x_2 + \Delta x_3 + \ldots + \Delta x_n = 10^6 \cdot 10^{-7} = 0.1$

$\varepsilon_S = \frac{\Delta_S}{\bar{S}} = \frac{0.1}{2.1325623} = 4.69 \cdot 10^{-2}$

5. $F = \dfrac{x+y^2}{xy - \frac{x}{y} + \frac{y}{2}}$, $\Delta x = \Delta y = 0.02$, $\bar{x} = 1$, $\bar{y} = 2$

$\varepsilon_F = ?$, $\Delta_F = ?$, $\bar{F} = ?$

$$\varepsilon_F = \varepsilon_{x+y^2} + \varepsilon_{xy-\frac{x}{y}+\frac{y}{2}}$$

$$\varepsilon_{x+y^2} = \frac{\Delta_{x+y^2}}{|\bar{x}+\bar{y}^2|}$$

$$\Delta_{x+y^2} = \Delta x + \Delta y^2 = \Delta x + \varepsilon_{y^2} \cdot |\bar{y}^2|$$

$$\Delta_{x+y^2} = \Delta x + (\varepsilon_y + \varepsilon_y) \cdot |\bar{y}^2| = 0.02 + 2 \cdot \frac{\Delta y}{\bar{y}} \cdot |\bar{y}^2| = 0.02 + 2 \cdot \frac{0.02}{2} \cdot 2^2$$

$$\Delta_{x+y^2} = 0.02 + 4 \cdot 0.02 = 0.02 + 0.08 = 0.1$$

$$\varepsilon_{x+y^2} = \frac{0.1}{|1+2^2|} = \frac{0.1}{5} = 0.02$$

$$\varepsilon_{xy-\frac{x}{y}+\frac{y}{2}} = \frac{\Delta_{xy-\frac{x}{y}+\frac{y}{2}}}{|\bar{x}\bar{y} - \frac{\bar{x}}{\bar{y}} + \frac{\bar{y}}{2}|}$$

$$\Delta_{xy-\frac{x}{y}+\frac{y}{2}} = \Delta_{xy} + \Delta_{\frac{x}{y}} + \Delta_{\frac{y}{2}}$$

$$\Delta_{xy} = \varepsilon_{xy} \cdot |\bar{x}\bar{y}| = (\varepsilon_x + \varepsilon_y) \cdot |\bar{x} \cdot \bar{y}|$$

$$\left.\begin{aligned} \varepsilon_x &= \frac{\Delta x}{\bar{x}} = \frac{0.02}{1} = 0.02 \\ \varepsilon_y &= \frac{\Delta y}{\bar{y}} = \frac{0.02}{2} = 0.01 \end{aligned}\right\} \Rightarrow \Delta_{xy} = (0.02 + 0.01)|1 \cdot 2| = 0.06$$

$$\Delta_{\frac{x}{y}} = \left|\frac{\bar{x}}{\bar{y}}\right| \cdot \varepsilon_{\frac{x}{y}} = \left|\frac{\bar{x}}{\bar{y}}\right| (\varepsilon_x + \varepsilon_y) = \frac{1}{2} \cdot 0.03 = 0.015$$

$$\Delta_{\frac{y}{2}} = \left|\frac{\bar{y}}{2}\right| \cdot \varepsilon_{y/2} = \left|\frac{\bar{y}}{2}\right| \cdot \varepsilon_y = 0.01 \cdot 1 = 0.01$$

$$\Delta_{xy-\frac{x}{y}+\frac{y}{2}} = 0.06 + 0.015 + 0.01 = 0.085$$

$$\varepsilon_{xy-\frac{x}{y}+\frac{y}{2}} = \frac{\Delta_{xy-\frac{x}{y}+\frac{y}{2}}}{\left|\bar{x}\bar{y} - \frac{\bar{x}}{\bar{y}} + \frac{\bar{y}}{2}\right|} = \frac{0.085}{\left|1 \cdot 2 - \frac{1}{2} + \frac{2}{2}\right|} = \frac{0.085}{\frac{5}{2}} = 0.034$$

$$\varepsilon_F = 0.02 + 0.034 = 0.054$$

$$\bar{F} = \frac{\bar{x} + \bar{y}^2}{\bar{x}\bar{y} - \frac{\bar{x}}{\bar{y}} + \frac{\bar{y}}{2}} = \frac{1+4}{2 - \frac{1}{2} + 1} = \frac{5}{\frac{5}{2}} = 2$$

$$\Delta_F = |\bar{F}| \cdot \varepsilon_F = 2 \cdot 0.054 = 0.108$$