Kretanje tela u gravitacionom polju Zemlje

2. Njutnov zakon

$$v(t) = \frac{dy(t)}{dt}$$

$$a(t) = \frac{dv(t)}{dt}$$

$$a(t) = \frac{1}{m} F(y, v, t)$$

$$\frac{d^2 y(t)}{dt^2} = \frac{F(y, v, t)}{m}$$

Slobodni pad i vertikalni hitac

$$v(t) = v_0 + g \cdot t$$

$$y(t) = y_0 + v_0 \cdot t + \frac{1}{2} g \cdot t^2$$

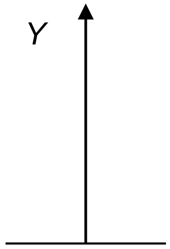
$$v(t) = v_0 - g \cdot t$$

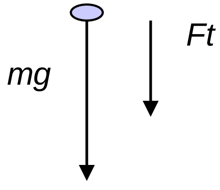
$$y(t) = y_0 + v_0 \cdot t - \frac{1}{2} g \cdot t^2$$

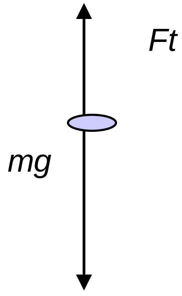
Realnija situacija

$$F_g = \frac{\gamma \cdot M \cdot m}{(R+y)^2} = \frac{\gamma \cdot M \cdot m}{R^2(1+\frac{y}{R})^2} = \frac{g \cdot m}{(1+\frac{y}{R})^2} = mg(1-2\frac{y}{R}+\ldots)$$

$$F = F_g - F_t = m \cdot g - F_t$$







Sila otpora vazduha

$$F_t(v) = k_1 \cdot v$$

$$F_t(v) = k_2 \cdot v^2$$

$$F_g = F_t$$

$$m \cdot g = k_1 \cdot v_1 \Rightarrow v_1 = \frac{m \cdot g}{k_1}$$

$$v_2 = \sqrt{\frac{m \cdot g}{k_2}}$$

$$F_t(v) = k_1 \cdot v_1 \cdot \left(\frac{v}{v_1}\right) = mg \cdot \left(\frac{v}{v_1}\right)$$

$$F_t(v) = k_2 \cdot {v_2}^2 \cdot \left(\frac{v}{v_2}\right)^2 = mg \cdot \left(\frac{v}{v_2}\right)^2$$

$$F_1(v) = m \cdot g \cdot \left(1 - \frac{v}{v_1}\right)$$

$$F_2(v) = m \cdot g \cdot \left(1 - \frac{v^2}{{v_2}^2}\right)$$

Numeričko rešenje jednačine kretanja

$$t_n = t_0 + n \cdot \Delta t$$

$$v_{n+1} = v_n + a_n \cdot \Delta t$$

$$y_{n+1} = y_n + v_n \cdot \Delta t$$

Metod Ojler-Kromera

$$v_{n+1} = v_n + a_n \cdot \Delta t$$

$$y_{n+1} = y_n + v_n \cdot \Delta t$$

Ojlerov metod

$$v_{n+1} = v_n + a_n \cdot \Delta t$$

$$y_{n+1} = y_n + v_{n+1} \cdot \Delta t$$

Metod Ojler Kromera

Program fall

```
 # include <stdio.h>
# include <math.h>


main ()
{

        FILE *fp1;
        fp1=fopen("pad.data","w");
        double y,v,a,g,t,dt;

        t=0;
        y=10;
        v=0;
        g=9.81;
        dt=0.001;
```

```
while (y >= 0)
{

a=g;
v=v+a*dt;
y=y-v*dt;
t=t+dt;
fprintf(fp1,"%10.3f,%10.3f,%10.3
f,%12.3f,\n",t,y,v,a);

}
fclose(fp1);
}
```

Euler-Richardson metod

$$a_n = F(y_n, v_n, t_n)/m,$$

$$v_{mid} = v_n + \frac{1}{2} a_n \Delta t,$$

$$y_{mid} = y_n + \frac{1}{2} v_n \Delta t,$$

$$a_{mid} = F(y_{mid}, v_{mid}, \frac{1}{2}\Delta t)/m,$$

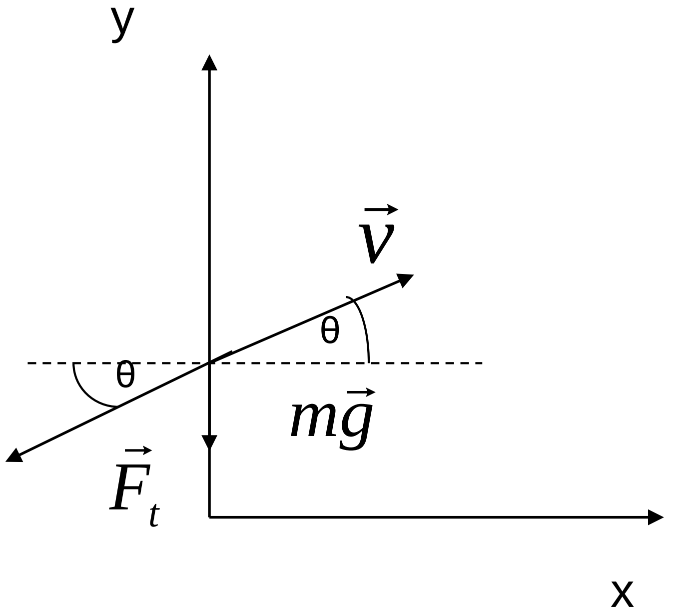
$$v_{n+1} = v_n + a_{mid}\Delta t,$$

$$y_{n+1} = y_n + v_{mid}\Delta t$$

Tabela eksperimentalnih rezultata

Vreme (s)	Koordinata (m)
-0.132	0.0
0.0	0.075
0.1	0.260
0.2	0.525
0.3	0.870
0.4	1.27
0.5	1.73
0.6	2.23
0.7	2.77
0.8	3.35

Dvodimenzione trajektorije



$$m \cdot \frac{dv_x}{dt} = -F_t \cdot \cos\Theta$$

$$m \cdot \frac{dv_y}{dt} = -mg - F_t \cdot \sin\Theta$$

$$F_t \approx k_2 v^2$$

$$v_x = v \cdot \cos\Theta$$

$$v_y = v \cdot \sin\Theta$$

$$\frac{dv_x}{dt} = -A \cdot v \cdot v_x$$

$$\frac{dv_y}{dt} = -g - A \cdot v \cdot v_y$$

$$A = \frac{k_2}{m}$$