RAVNE POVRSI JEDNO I GLAVNE OSE INERCIJE.

AKO SE GLAVNE OSE INERCIJE ODNOSE NA TEŽIŠTE RAVNE POVRSI ONDA SE RADI O **GLAVNIM TEZIŠNIM OSAMA INERCIJE** ODNOSNO O **GLAVNIM TEZIŠNIM MOMENTIMA INERCIJE**.

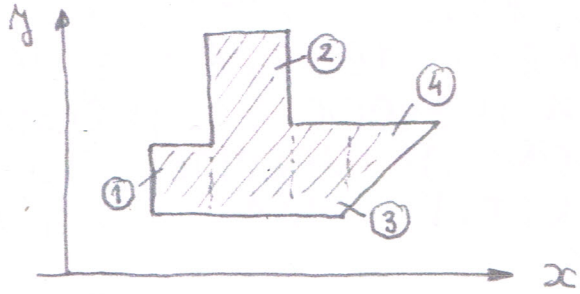
- MOMENTI INERCIJE SLOZENE RAVNE POVRSI ZA NEKE OSE JEDNAKI SU ZBIRU MOMENATA INERCIJE JEDNOSTAVNIH RAVNIH POVRSI ZA RAZMATRANE OSE TJ. (SL.)

$$I_x = I_x^{(1)} + I_x^{(2)} + \dots = \sum_{i=1}^{n} I_x^{(i)}$$

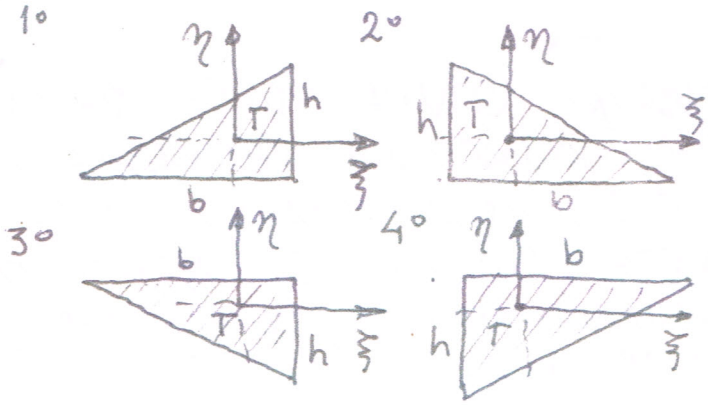
$$I_y = I_y^{(1)} + I_y^{(2)} + \dots = \sum_{i=1}^{n} I_y^{(i)} \qquad \dots (9)$$

$$I_{xy} = I_{xy}^{(1)} + I_{xy}^{(2)} + \dots = \sum_{i=1}^{n} I_{xy}^{(i)}$$

OVU OSOBINU NECEMO DOKAZIVATI.



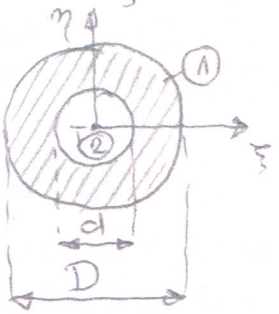
PRIMJER: ZA TROUGLOVE DIMENZIJA b i h ODREDITI ZNAK CENTRIFUGALNOG MOMENTA INERCIJE ZA TEŽIŠNE OSE ξ i η. (SL).



IMAJUĆI U VIDU OSOBINU MOMENATA INERCIJE NIJE TEŠKO ZAKLJUČITI DA JE:

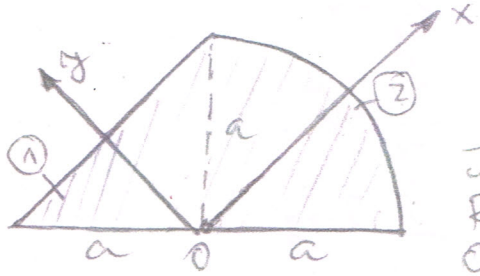
1° $I_{\xi\eta} > 0$; 3° $I_{\xi\eta} < 0$
2° $I_{\xi\eta} < 0$; 4° $I_{\xi\eta} > 0$

PRIMJER: ZA RAVNU POVRŠ SA SLIKE NACI MOMENT INERCIJE I_ξ. DATO JE D i d



$$I_\xi = I_\xi^{①} - I_\xi^{②} = \frac{\pi D^4}{64} - \frac{\pi d^4}{64} = \frac{\pi}{64}(D^4 - d^4)$$

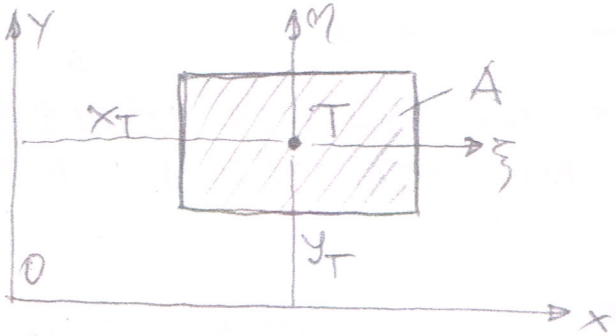
PRIMJER: ZA RAVNU POVRŠ SA SLIKE ODREDITI CENTRIFUGALNI MOMENT INERCIJE I_{xy}.



$$I_{xy} = I_{xy}^{①} + I_{xy}^{②} = 0 + 0 = 0,$$

JER JE OSA x OSA SIMETRIJE RAVNE POVRSI ② A OSA y JE OSA SIMETRIJE RAVNE POVRSI ①.

- ŠTAJNER-ova TEOREMA: MOMENTI INERCIJE RAVNE POVRSI ZA OSE KOJE SU PARALELNO POMJERENE U ODNOSU NA MJENE TEZISNE OSE JEDNAKI SU ZBIRU MOMENATA INERCIDE ZA TEZISNE OSE I_ξ, I_η i $I_{\xi\eta}$ (SOPSTVENI MOMENTI INERCIJE), TJ. POLOZAJNIH MOMENATA INERCIJE $y_T^2 A$ i $x_T^2 A$ i $x_T y_T A$, TJ.

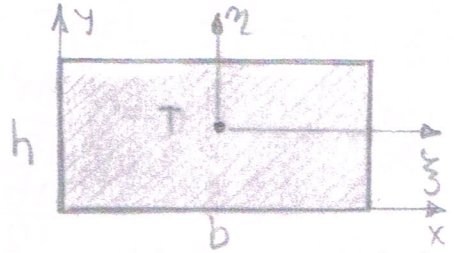


$$I_x = I_\xi + y_T^2 A$$
$$I_y = I_\eta + x_T^2 A \qquad (10)$$
$$I_{xy} = I_{\xi\eta} + x_T y_T A.$$

PRIMJER: ZA JEDNOSTAVNE RAVNE POVRSI PRIKAZANE NA SLICI, PRIMJENOM STAJNEROVE TEOREME IZVESTI IZRAZE ZA ODREDIVANJE MOMENATA INERCIJE ZA NAZNACENE KARAKTERISTICNE OSE, AKO SU POZNATI IZRAZI ZA ODREDIVANJE MOMENATA INERCIJE ZA TEZISNE OSE. DATO JE: a, b i h.

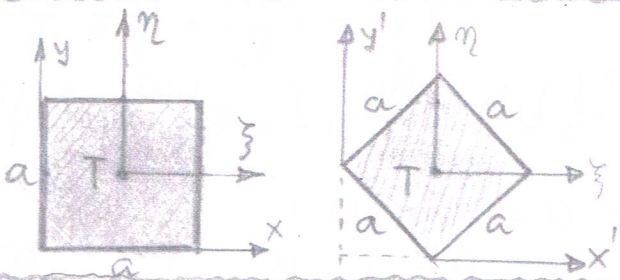
TABLICA

MOMENTA INERCIJE JEDNOSTAVNIH RAVNIH POVRŠI



$$I_\xi = \frac{bh^3}{12}; \quad I_\eta = \frac{hb^3}{12}; \quad I_{\xi\eta} = 0$$

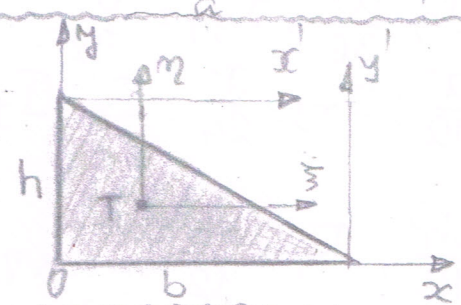
$$I_x = \frac{bh^3}{3}; \quad I_y = \frac{hb^3}{3}; \quad I_{xy} = \frac{b^2h^2}{4}$$



$$I_\xi = I_\eta = \frac{a^4}{12}; \quad I_{\xi\eta} = 0$$

$$I_x = I_y = \frac{a^4}{3}; \quad I_{xy} = \frac{a^4}{4}$$

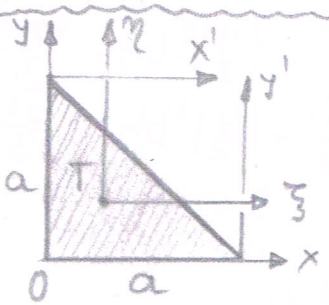
$$I_{x'} = I_{y'} = \frac{7}{12}a^4$$



$$I_\xi = \frac{bh^3}{36}; \quad I_\eta = \frac{hb^3}{36}; \quad I_{\xi\eta} = \pm\frac{b^2h^2}{72}$$

$$I_x = \frac{bh^3}{12}; \quad I_y = \frac{hb^3}{12}; \quad I_{xy} = \pm\frac{b^2h^2}{24}$$

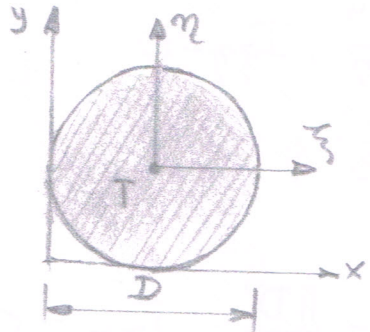
$$I_{x'} = \frac{bh^3}{4}; \quad I_{y'} = \frac{hb^3}{4}$$



$$I_\xi = I_\eta = \frac{a^4}{36}; \quad I_{\xi\eta} = \pm\frac{a^4}{72}$$

$$I_x = I_y = \frac{a^4}{12}; \quad I_{xy} = \frac{a^4}{24};$$

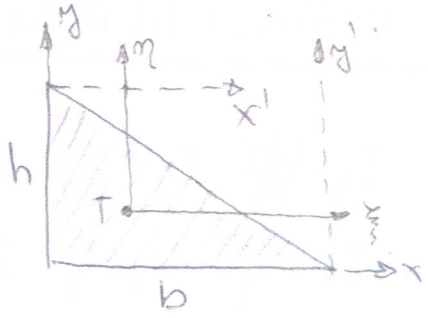
$$I_{x'} = I_{y'} = \frac{a^4}{4}$$



$$I_\xi = I_\eta = \frac{\pi D^4}{64}; \quad I_{\xi\eta} = 0$$

$$I_x = I_y = \frac{5}{64}\pi D^4; \quad I_{xy} = \frac{\pi D^4}{16}$$

1°



$I_\xi = \frac{bh^3}{36}$; $I_\eta = \frac{hb^3}{36}$; $I_{\xi\eta} = -\frac{b^2h^2}{72}$

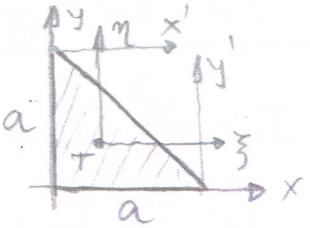
$I_x = \frac{bh^3}{36} + \left(\frac{h}{3}\right)^2 \cdot \frac{1}{2} bh = \frac{bh^3}{12}$

$I_y = \ldots\ldots\ldots = \frac{hb^3}{12}$

$I_{xy} = -\frac{b^2h^2}{72} + \frac{h}{3}\frac{b}{3}\frac{bh}{2} = \frac{b^2h^2}{24}$

$I_{y'} = \frac{hb^3}{36} + \left(\frac{2}{3}b\right)^2 \cdot \frac{1}{2} bh = \frac{b^3h}{4}$; $I_{x'} = \frac{bh^3}{36} + \left(\frac{2}{3}h\right)^2 \cdot \frac{1}{2} bh = \frac{bh^3}{4}$

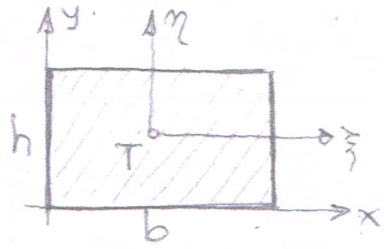
SPECIJALNO ZA $a = b = h$ VAŽI



$I_\xi = \frac{a^4}{36}$; $I_\eta = \frac{a^4}{36}$; $I_{\xi\eta} = -\frac{a^4}{72}$

$I_x = I_y = \frac{a^4}{12}$; $I_{xy} = \frac{a^4}{24}$ $I_{x'} = I_{y'} = \frac{a^4}{4}$

2°



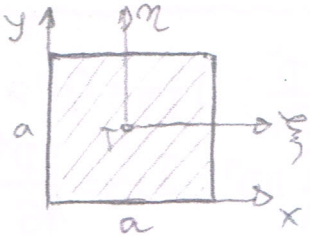
$I_\xi = \frac{bh^3}{12}$; $I_\eta = \frac{hb^3}{12}$; $I_{\xi\eta} = 0$.

$I_x = \frac{bh^3}{12} + \left(\frac{h}{2}\right)^2 bh = \frac{bh^3}{3}$

$I_y = \frac{hb^3}{12} + \left(\frac{b}{2}\right)^2 bh = \frac{b^3h}{3}$

$I_{xy} = 0 + \frac{b}{2} \cdot \frac{h}{2} bh = \frac{b^2h^2}{4}$;

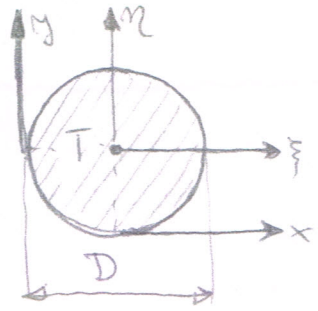
SPECIJALNO, ZA $a = b = h$ VAŽI;



$I_\xi = I_\eta = \frac{a^4}{12}$; $I_{\xi\eta} = 0$

$I_x = I_y = \frac{a^4}{3}$

3°

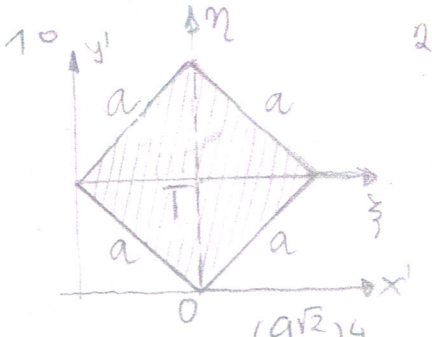


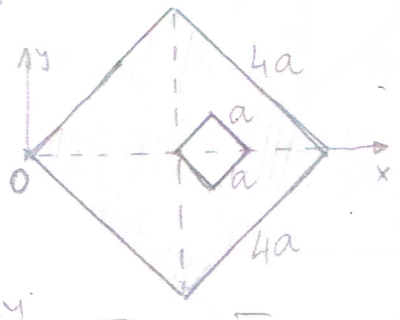
$I_\xi = I_\eta = \frac{\pi D^4}{64}$, $I_{\xi\eta} = 0$

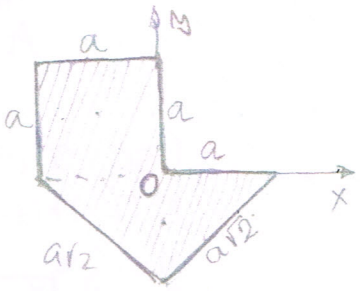
$I_x = \frac{\pi D^4}{64} + \left(\frac{D}{2}\right)^2 \frac{\pi D^2}{4} = \frac{5}{64}\pi D^4 = I_y$

$I_{xy} = 0 + \frac{D}{2} \cdot \frac{D}{2} \frac{\pi D^2}{4} = \frac{\pi D^4}{16}$

PRIMJER: ZA SLOŽENE RAVNE POVRSI PRIKAZANE NA SLICI ODREDITI MOMENTE INERCIJE ZA OSE x i y. POTREBNI PODACI SU DATI NA SLICI.

1° 

2° 

3° 

1° $I_\xi = 4 \cdot \frac{(\frac{a\sqrt{2}}{2})^4}{12} = \frac{a^4}{12} = I_\eta \; ; \; I_{\xi\eta} = 0$

$I_{x'} = I_x + (\frac{a\sqrt{2}}{2})^2 a^2 = \frac{a^4}{12} + \frac{a^4}{2} = \frac{7}{12}a^4 = I_{y'}$

2° $I_x = \frac{(4a)^4}{12} - \frac{a^4}{12} = \frac{83}{4}a^4$

$I_y = \left[\frac{(4a)^4}{12} + (2a\sqrt{2})^2 \cdot 16a^2\right] - \left[\frac{a^4}{12} + (\frac{5a\sqrt{2}}{2})^2 a^2\right] = \frac{547}{4}a^4$

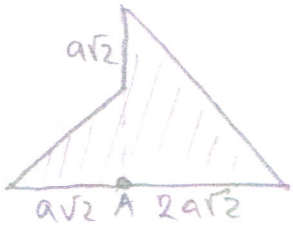
$I_{xy} = 0$

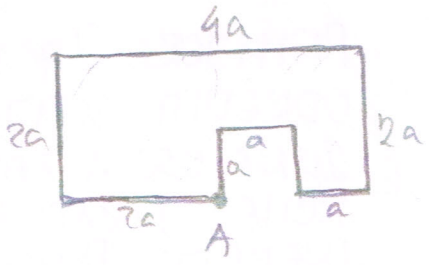
3° $I_x = \frac{a^4}{3} + \frac{1}{2}\frac{(a\sqrt{2})^4}{12} = \frac{a^4}{2}$

$I_y = \frac{a^4}{3} + 2\frac{a^4}{12} = \frac{a^4}{2}$

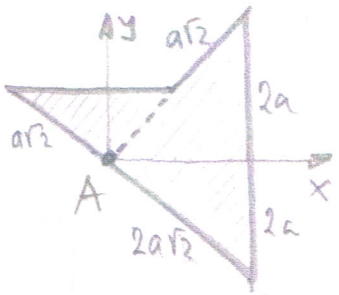
$I_{xy} = 0 + (-\frac{a^4}{4}) = -\frac{a^4}{4}$

PRIMJER: ZA SLOŽENE RAVNE POVRSI PRIKAZANE NA SLICI ODREDITI U TAČKI A POLOŽAJ OSA ZA KOJE JE CENTRIFUGALNI MOMENT INERCIJE JEDNAK NULI (GLAVNE OSE INERCIJE) I IZRAČUNATI VRIJEDNOSTI AKSIJALNIH MOMENATA INERCIJE (GLAVNI MOMENTI INERCIJE). POTREBNI PODACI SU DATI NA SLICI.

1° 

2° 

1°



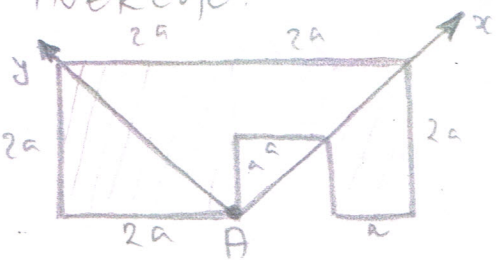
$$I_x = 2\cdot\frac{(2a)^4}{12} + 2\,\frac{a^4}{4} = \frac{19}{6}a^4 \;//$$

$$I_y = 2\cdot\frac{a^4}{12} + 2\,\frac{(2a)^4}{4} = \frac{49}{6}a^4 \;//$$

$$I_{xy} = 0$$

OSE x i y SU GLAVNE OSE a I_x i I_y SU GLAVNI MOMENTI INERCIJE.

2°



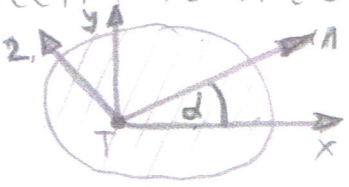
$$I_x = \frac{(2a)^4}{12} - \frac{a^4}{12} + \frac{7}{12}(2a)^4 = \frac{127}{12}a^4$$

$$I_y = \frac{(2a)^4}{12} + \frac{7}{12}(2a)^4 - \frac{7}{12}a^4 = \frac{121}{12}a^4$$

$$I_{xy} = 0$$

OSE x i y SU GLAVNE OSE INERCIJE, a I_x i I_y SU GLAVNI MOMENTI INERCIJE.

• POLOŽAJ GLAVNIH OSA INERCIJE RAVNE POVRŠI TJ. OSA ZA KOJE JE CENTRIFUGALNI MOMENT INERCIJE JEDNAK NULI, ZA SLUČAJ KADA RAVNA POVRŠ NEMA OSA SIMETRIJE ODREĐEN JE UGLOM α KOJI JE ODREĐEN SLJEDEĆIM IZRAZOM



$$\alpha = \frac{1}{2}\operatorname{arctg}\left(-\frac{2I_{xy}}{I_x - I_y}\right) \qquad (11)$$

VRIJEDNOSTI MOMENATA INERCIJE ZA GLAVNE OSE 1 i 2 ODREĐENE SU IZRAZIMA

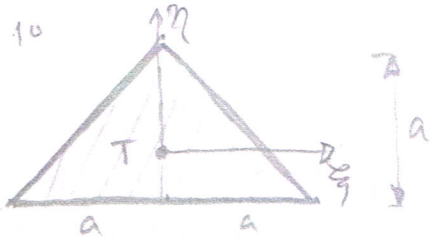
$$I_1, I_2 = \frac{1}{2}(I_x + I_y) \pm \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2} \qquad (12)$$

PRI ČEMU VAŽI DA

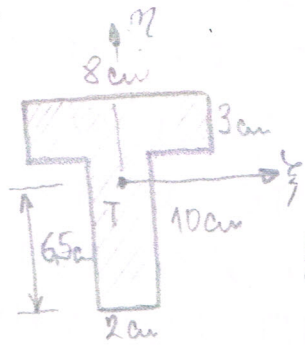
- AKO JE $I_x > I_y$ $\quad x \xrightarrow{\alpha} 1$; $y \xrightarrow{\alpha} 2$
- AKO JE $I_x < I_y$ $\quad x \xrightarrow{\alpha} 2$; $y \xrightarrow{\alpha} 1$

PRIMJER: ZA RAVNE POVRŠI PRIKAZANE NA SLICI ODREDITI VRIJEDNOSTI MOMENATA INERCIJE ZA TEŽIŠNE OSE A ZATIM NAĆI POLOŽAJ GLAVNIH OSA INERCIJE I IZRAČUNATI VRIJEDNOSTI GLAVNIH MOMENATA INERCIJE. DATO JE a. POLOŽAJ TEŽIŠTA JE POZNAT ZA SLUČAJEVE 1° i 2°.

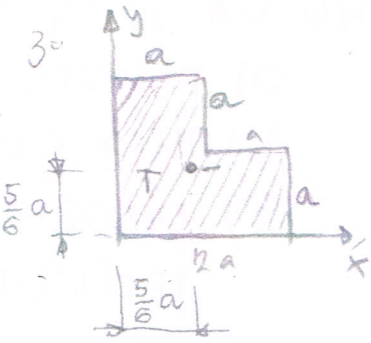
1°



2°

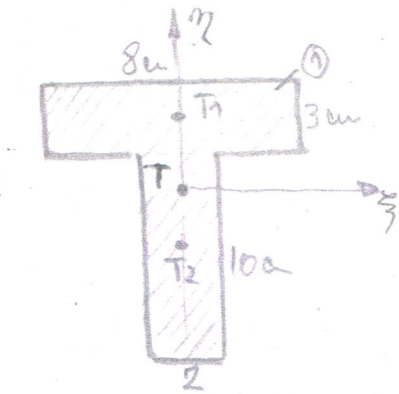


3°



1° $I_\xi = 2 \cdot \frac{a^4}{36} = \frac{a^4}{18}$; $I_\eta = 2\,\frac{a^4}{12} = \frac{a^4}{6}$; $I_{\xi\eta} = 0$; ξ, η GLAVNE OSE.

2°



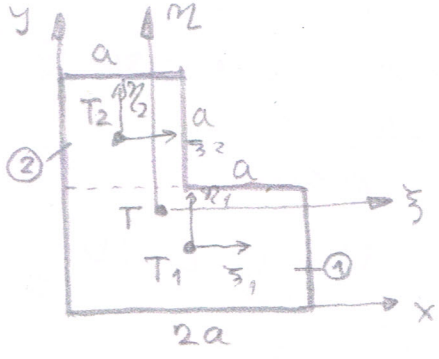
$I_\xi = \left(\frac{8 \cdot 3^3}{12} + 5^2 \cdot 3 \cdot 4\right) + \left(2\,\frac{10^3}{12} + 1{,}5^2 \cdot 2 \cdot 10\right) = 646\,cm^4$

$I_\eta = \frac{10\,2^3}{12} + \frac{3 \cdot 8^3}{12} = 134{,}66\,cm^4$

$I_{\xi\eta} = 0$

OSE ξ i η SU GLAVNE OSE.

3° R: $I_\xi = \frac{11}{12}a^4$; $I_\eta = \frac{11}{12}a^4$; $I_{\xi\eta} = -\frac{a^4}{3}$; ξ, η GLAVNE OSE.



T_1: $x_{T_1} = a$; $y_{T_1} = \frac{a}{2}$; $A_1 = 2a^2$

T_2: $x_{T_2} = \frac{a}{2}$; $y_{T_2} = \frac{3}{2}a$; $A_2 = a^2$

$x_T = \frac{a \cdot 2a^2 + \frac{a}{2}a^2}{a^2 + 2a^2} = \frac{5}{6}a$

$y_T = \frac{\frac{a}{2}2a^2 + \frac{3}{2}a \cdot a^2}{2a^2 + a^2} = \frac{5}{6}a.$

T_1: $\xi_{T_1} = \frac{a}{6}$; $\eta_{T_1} = -\frac{1}{3}a$; $A_1 = 2a^2$; $I_{\xi_1} = \frac{2a^4}{6}$; $I_{\eta_1} = \frac{2}{3}a^4$; $I_{\xi_1\eta_1} = 0$

T_2: $\xi_{T_2} = -\frac{a}{3}$; $\eta_{T_2} = \frac{4}{6}a$; $A_2 = a^2$; $I_{\xi_2} = \frac{a^4}{12} = I_{\eta_2}$; $I_{\xi_2\eta_2} = 0$

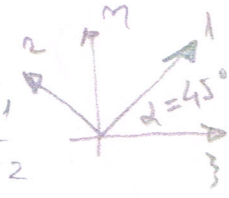
$I_\xi = (I_{\xi_1} + \eta_{T_1}^2 A_1) + (I_{\xi_2} + \eta_{T_2}^2 A_2) = \dots = \frac{11}{12}a^4$

$I_\eta = (I_{\eta_1} + \xi_{T_1}^2 A_1) + (I_{\eta_2} + \xi_{T_2}^2 A_2) = \dots = \frac{11}{12}a^4$

$I_{\xi\eta} = (I_{\xi_1\eta_1} + \xi_{T_1}\eta_{T_1}A_1) + (I_{\xi_2\eta_2} + \xi_{T_2}\eta_{T_2}A_2) = \dots = -\frac{a^4}{3}$

$\alpha = \frac{1}{2}\,arctg\left(-\frac{2I_{\xi\eta}}{I_\xi - I_\eta}\right) = \frac{1}{2}\,arctg\,\infty = 45^\circ$

$I_1, I_2 = \frac{1}{2}(I_\xi + I_\eta) \pm \frac{1}{2}\sqrt{(I_\xi - I_\eta)^2 + 4I_{\xi\eta}^2} = \begin{cases} \frac{15}{12}a^4 = I_1 \\ \frac{7}{12}a^4 = I_2 \end{cases}$

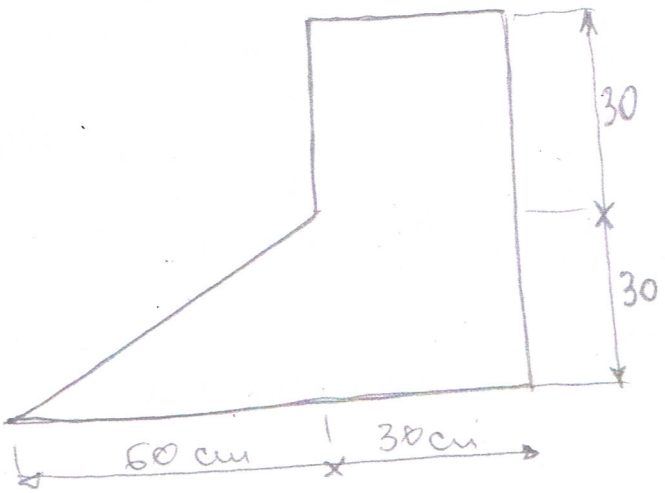


PRIMJER: ZA SLOŽENU RAVNU DOVRŠ PRIKAZANO NA SLICI ODREDITI:

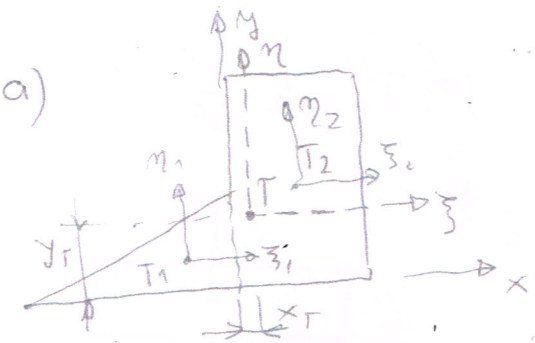
a) POLOŽAJ TEŽIŠTA

b) MOMENTE INERCIJE ZA TEŽIŠNE OSE

c) POLOŽAJ GLAVNIH OSA INERCIJE I VRIJEDNOSTI GLAVNIH MOMENATA INERCIJE



a)



T_1: $x_{T_1} = -20$; $y_{T_1} = 10$; $A_1 = 900\,cm^2$

T_2: $x_{T_2} = 15$; $y_{T_2} = 30$; $A_2 = 1800\,cm^2$

$$x_T = \frac{x_{T_1}A_1 + x_{T_2}A_2}{A_1 + A_2} = \ldots = 3{,}33\,cm$$

$$y_T = \frac{y_{T_1}A_1 + y_{T_2}A_2}{A_1 + A_2} = \ldots = 23{,}33\,cm$$

b) KOORDINATE TEŽIŠTA U ODNOSU NA OSE ξ, η SU:

T_1: $\xi_{T_1} = -23{,}3\,cm$ $\eta_{T_1} = -13{,}3\,cm$ $A_1 = 900\,cm^2$

T_2: $\xi_{T_2} = 1{,}17\,cm$ $\eta_{T_2} = 6{,}7\,cm$ $A_2 = 1800\,cm^2$

MOMENTI INERCIJE ZA TEŽIŠNE OSE SU

$$I_{\xi_1} = \frac{60 \cdot 30^3}{36}\,; \quad I_{\eta_1} = \frac{30\,60^3}{36}\,; \quad I_{\xi_1\eta_1} = \frac{30^2 \cdot 60^2}{72}$$

$$I_{\xi_2} = \frac{30 \cdot 60^3}{12}\,; \quad I_{\eta_2} = \frac{60 \cdot 30^3}{12}\,; \quad I_{\xi_2\eta_2} = 0$$

PRIMJENOM STAJNEROVE TEOREME SE DOBIJA

$$I_\xi = \ldots = 82{,}5 \cdot 10^4\,cm^4$$

$$I_\eta = \ldots = 105 \cdot 10^4\,cm^4$$

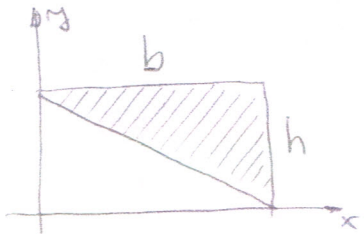
$$I_{\xi\eta} = \ldots = 46{,}5 \cdot 10^4\,cm^4$$

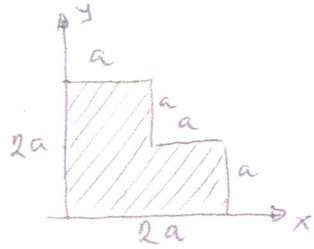
c) $$\alpha = \frac{1}{2}\,arctg\left(-\frac{2 \cdot 46{,}5}{82{,}5 - 105}\right) = 38{,}2°$$

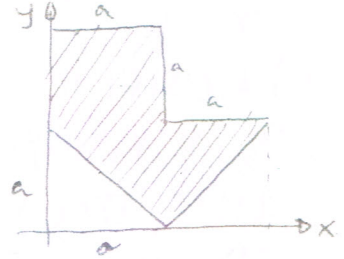
$$I_1, I_2 = \frac{1}{2}(82{,}5 + 105) \pm \frac{1}{2}\sqrt{(82{,}5 - 105)^2 + 4 \cdot 46{,}5^2} = \begin{cases} 141{,}59\,cm^4 = I_1 \\ 45{,}91\,cm^4 = I_2 \end{cases}$$

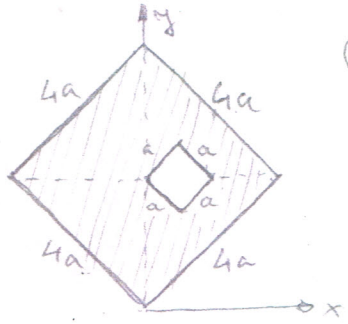
ZADACI ZA VJEŽBANJE

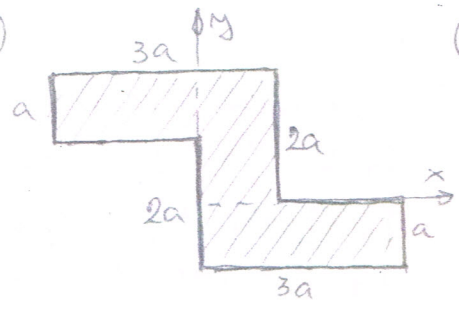
• PRIMJERI 1. ZA RAVNE POVRSI PRIKAZANE NA SLICI ODREDITI MOMENTE INERCIJE ZA OSE PRIKAZANE NA SLICI. DATO JE a, h, b

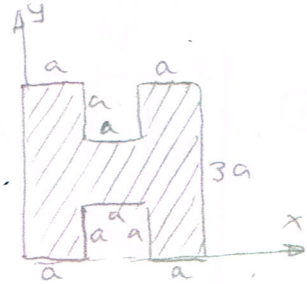
① 

② 

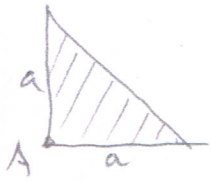
③ 

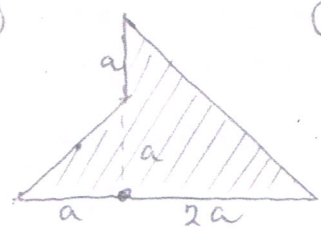
④ 

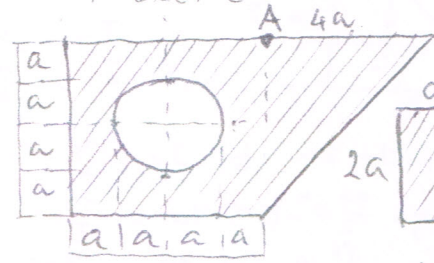
⑤ 

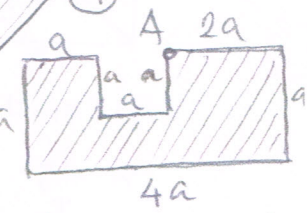
⑥ 

• PRIMJERI 2. ZA RAVNE POVRSI PRIKAZANE NA SLICI ODREDITI POLOZAJ GLAVNIH OSA INERCIJE UTACKIA A ZATIM IZRACUNATI VRIJEDNOST JEDNOG OD DVA GLAVNA MOMENTA INERCIJE. DATO JE a.

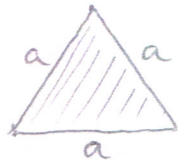
① 

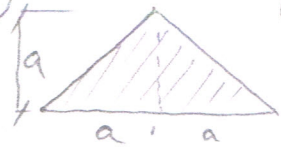
② 

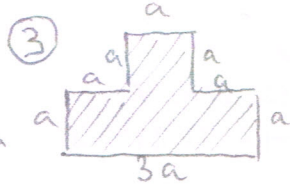
③ 

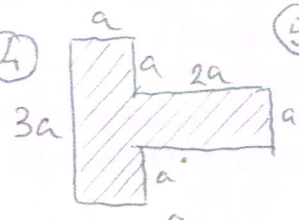
④ 

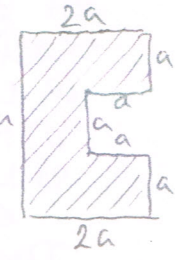
• PRIMJERI 3. ZA RAVNE POVRSI PRIKAZANE NA SLICI ODREDITI POLOZAJ TEZISTA A ZATIM IZRACUNATI VRIJEDNOSTI MOMENATA INERCIJE ZA TEZISNE OSE. DATO JE a.

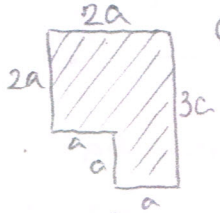
① 

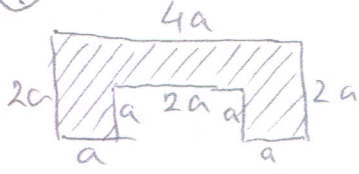
② 

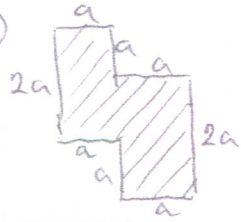
③ 

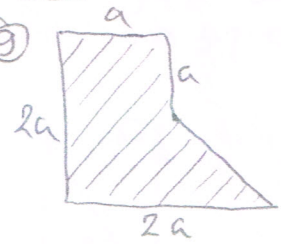
④ 

⑤ 

⑥ 

⑦ 

⑧ 

⑨ 

RJEŠENJA PR 1 ① $\frac{bh^3}{4}$; $\frac{b^3h}{4}$; $\frac{b^2h^2}{24}$ ② $3a^4$; $3a^4$; $\frac{7}{4}a^4$; ③ $\frac{47}{18}a^4$; $\frac{27}{18}a^4$; $\frac{67}{36}a^4$; ④ $\frac{563}{4}a^4$; $\frac{83}{4}a^4$; $-2a^4$; ⑤ $\frac{25}{3}a^4$; $\frac{37}{3}a^4$; $-\frac{17}{4}a^4$; ⑥ $\frac{61}{3}a^4$; $\frac{25}{3}a^4$; $\frac{21}{4}a^4$; PR 2 ① $\frac{a^4}{24}$; $\frac{a^4}{8}$; ② $\frac{19}{24}a^4$; $\frac{49}{24}a^4$; ④ $\frac{127}{12}a^4$; $\frac{121}{12}a^4$; PR 3 ① $\frac{13}{96}a^4$; $\frac{13}{32}a^4$; 0; ② $\frac{a^4}{18}$; $\frac{a^4}{6}$; 0 ③ $\frac{13}{12}a^4$ $\frac{7}{3}a^4$; 0; ④ $\frac{29}{12}a^4$; $\frac{4321}{150}a^4$; ⑤ $\frac{27}{6}a^4$; $\frac{97}{60}a^4$; ⑧ $\frac{7}{3}a^4$; $\frac{4}{3}a^4$; $-a^4$.

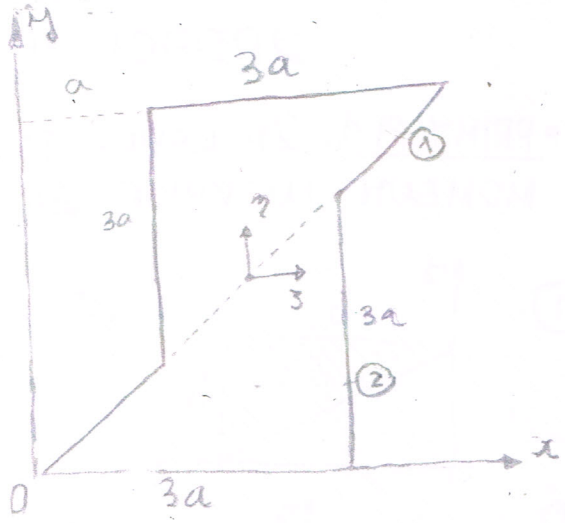
ZA RAVNU POVRŠ PRIKAZANU NA SLICI ODREDITI

a) POLOŽAJ TEŽIŠTA

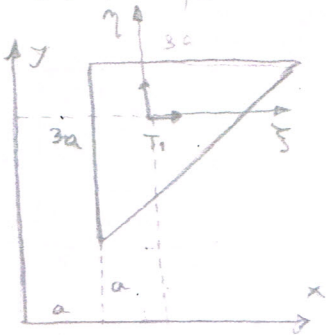
b) MOMENTE INERCIJE ZA OSE x, y

c) MOMENTE INERCIJE ZA OSE ξ IN KOJE PROLAZE KROZ TE ZISTE RAVNE PLOPŠI

Dano je a



①



$x_T = \frac{1}{2}a \quad y_T = 3a \quad A = \frac{9}{2}a^2$

$I_\xi = I_\eta = \frac{9}{4}a^4 \quad I_{\xi\eta} = \frac{9}{8}a^4$

$I_x = \frac{9}{4}a^4 + (3a)^2\frac{9}{2}a^2 = \frac{171}{4}a^4$

$I_y = \frac{9}{4}a^4 + (2a)^2\frac{9}{2}a^2 = \frac{81}{4}a^4$

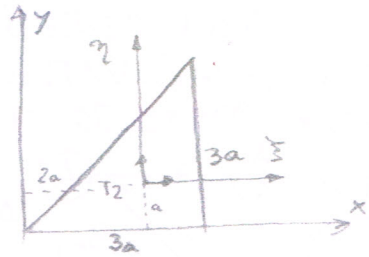
$I_{xy} = \frac{9}{8}a^4 + 2a\cdot 3a\,\frac{9}{2}a^2 = \frac{225}{8}a^4$

$I_\xi = \frac{3a\cdot(3a)^3}{36} = \frac{9}{4}a^4$ (TABLICE)

$I_{\xi\eta} = \frac{(3a)^2\cdot(3a)^2}{72} = \frac{9}{8}a^4$

$I_{xy} = I_{\xi\eta} + x_T\, y_T\, A$

②

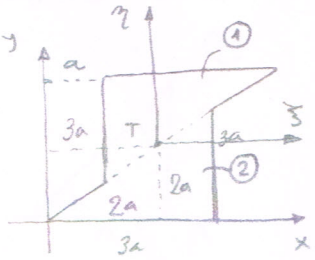


$x_T = 2a;\ y_T = a;\ A = \frac{9}{2}a^2 \quad I_\xi = I_\eta = \frac{9}{4}a^4;\ I_{\xi\eta} = +\frac{9}{8}a^4$ (TABLICE)

$I_x = \frac{9}{4}a^4 + a^2\frac{9}{2}a^2 = \frac{27}{4}a^4$

$I_y = \frac{9}{4}a^4 + (2a)^2\frac{9}{2}a^2 = \frac{81}{4}a^4$

$I_{xy} = \frac{9}{8}a^4 + 2a\cdot a\cdot\frac{9}{2}a^2 = \frac{81}{8}a^4$



a) $x_T = y_T = 2a$ b) $I_x = \frac{198}{4}a^4 = \frac{99}{2}a^4 \quad I_y = \frac{81}{2}a^4;\ I_{xy} = \frac{153}{4}a^4$

c) $I_\xi = \frac{99}{2}a^4 - (2a)^2\cdot 9a^2 = \frac{27}{2}a^4 \qquad I_\xi = I_x - y_T^2\cdot A$

$I_\eta = \frac{81}{2}a^4 - (2a)^2\cdot 9a^2 = \frac{9}{2}a^4 \qquad I_\eta = I_y - x_T^2\,A$

$I_{\xi\eta} = \frac{153}{4}a^4 - 2a\cdot 2a\cdot 9a^2 = \frac{153-144}{4}a^4 = \frac{9}{4}a^4 \qquad I_{\xi\eta} = I_{xy} - x_T\cdot y_T\cdot A$

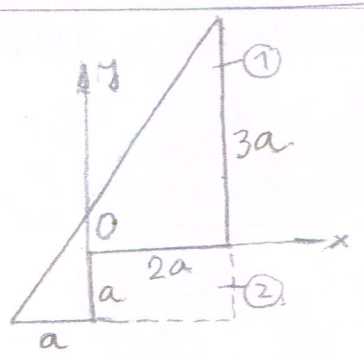
ZA RAVNU POVRŠ PRIKAZANU NA SLICI ODREDITI:

a) KOORDINATE TEŽIŠTA

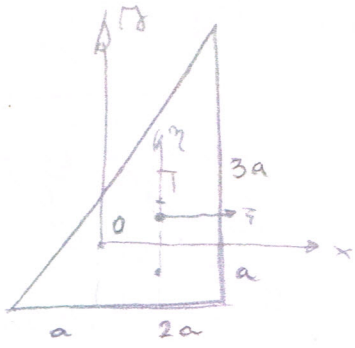
b) MOMENTE INERCIJE ZA OSE x, y

c) MOMENTE INERCIJE ZA TEŽIŠNE OSE ξ i η.

Dano je a.



①



$x_T = a \quad ; y_T = \frac{4a}{3} - a = \frac{a}{3} \quad ; A = \frac{1}{2}3a\cdot 4a = 6a^2$

$I_\xi = \frac{3a(4a)^3}{12\cdot 3} = \frac{16a^4}{3}$ /

$I_\eta = \frac{4a(3a)^3}{12\cdot 3} = \frac{9a^4}{3} = 3a^4$ /

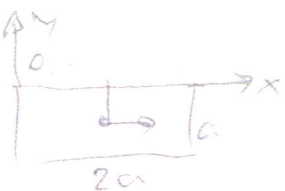
$I_{\xi\eta} = \frac{(3a)^2(4a)^2}{72} = 2a^4$ /

$I_x = \frac{16}{3}a^4 + \left(\frac{a}{3}\right)^2\cdot 6a^2 = 6a^4$ /

$I_y = 3a^4 + a^2\,6a^2 = 9a^4$ /

$I_{xy} = 2a^4 + a\cdot\frac{a}{3}\,6a^2 = 4a^4$ /

(2)



$x_T = a \quad y_T = -\frac{a}{2} \quad A = 2a^2$

$I_\xi = \frac{2a \cdot a^3}{12} = \frac{a^4}{6}$

$I_\eta = a \cdot \frac{12a^3}{12} = \frac{2}{3}a^4$

$I_{\xi\eta} = 0$

$I_x = \frac{a^4}{6} + \left(-\frac{a}{2}\right)^2 \cdot 2a^2 = \frac{2}{3}a^4$

$I_y = \frac{2}{3}a^4 + a^2 \cdot 2a^2 = \frac{8}{3}a^4$

$I_{xy} = 0 + a\left(-\frac{a}{2}\right)2a^2 = -a^4$ /

a) $x_T = \frac{a \cdot 6a^2 - a \cdot 2a^2}{4a^4} = a$ // $\quad y_T = \frac{\frac{a}{2} \cdot 6a^2 - \left(-\frac{a}{2}\right)2a^2}{4a^4} = \frac{2a^3 + a^3}{4a^2} = \frac{3}{4}a$ //

b) $I_x = 6a^4 - \frac{2}{3}a^4 = \frac{16}{3}a^4$

$I_y = 9a^4 - \frac{2}{3}a^4 = \frac{19}{3}a^4$

$I_{xy} = 4a^4 - (-a^4) = 5a^4$

c) $I_\xi = \frac{16}{3}a^4 - \left(\frac{3}{4}a\right)^2 4a^2 = \left(\frac{16}{3} - \frac{9}{4}\right)a^4 = \frac{37}{12}a^4$ /

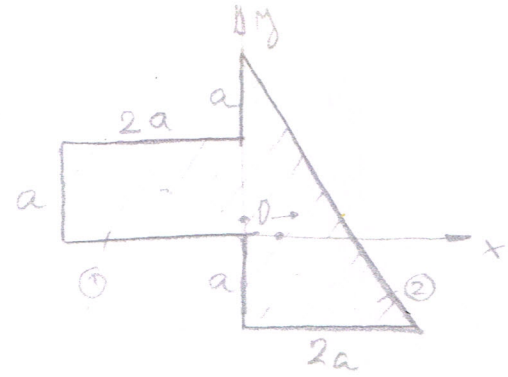
$I_\eta = \frac{19}{3}a^4 - a^2 \cdot 4a^2 = \frac{7}{3}a^4$ /

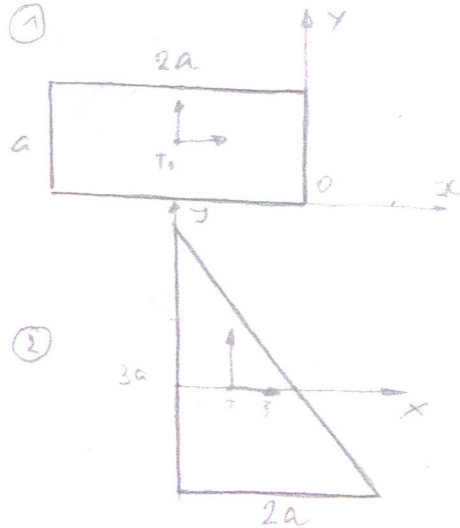
$I_{\xi\eta} = 5a^4 - (a)\left(\frac{3}{4}a\right)4a^2 = 2a^4$ /

ZA RAVNU POVRŠ PRIKAZANU NA SLICI ODREDITI:

a) POLOŽAJ TEŽIŠTA
b) MOMENTE INERCIJE ZA OSE x, y
c) MOMENTE INERCIJE ZA TEŽIŠNE OSE ξ, η.

Dato je a.





$x_T = -a \quad y_T = \frac{a}{2} \quad A = 2a^2$

$I_\xi = \frac{2a \cdot a^3}{12} = \frac{a^4}{6} \quad I_\eta = \frac{a \cdot (2a)^3}{12} = \frac{2}{3}a^4 \quad I_{\xi\eta} = 0$

$I_x = \frac{a^4}{6} + \left(\frac{a}{2}\right)^2 \cdot 2a^2 = \frac{a^4}{6} + \frac{a^4}{2} = \frac{2}{3}a^4$

$I_y = \frac{2}{3}a^4 + (-a)^2 2a^2 = \frac{2}{3}a^4 + 2a^4 = \frac{8}{3}a^4$

$I_{xy} = 0 + (-a)\left(\frac{a}{2}\right)2a^2 = -a^4$

$x_T = \frac{2a}{3} \quad y_T = 0 \quad A = \frac{1}{2} 2a\, 3a = 3a^2$

$I_\xi = \frac{2a(3a)^3}{36} = \frac{2 \cdot 3 \cdot 3 \cdot 3}{36}a^4 = \frac{3}{2}a^4 \quad I_\eta = \frac{3a(2a)^3}{36} = \frac{3 \cdot 2 \cdot 2 \cdot 2}{36}a^4 = \frac{2}{3}a^4$

$I_{\xi\eta} = -\frac{(2a)^2(3a)^2}{72} = -\frac{a^4}{2}$

$I_x = \frac{3}{2}a^4 + 0 \quad 3a^2 = \frac{3}{2}a^4$ / $\quad I_y = \frac{2}{3}a^4 + \left(\frac{2a}{3}\right)^2 3a^2 = \left(\frac{2}{3} + \frac{4 \cdot 3}{3}\right)a^4 = \frac{14}{3}a^4$ /

$I_{xy} = -\frac{a^4}{2} + 0 = -\frac{a^4}{2}$

a) $I_T = \frac{(-a)\,2a^2 + \frac{2a}{3} \cdot 3a^2}{2a^2 + 3a^2} = 0 \quad y_T = \frac{\frac{a}{2} \cdot 2a^2 + 0 \cdot 3a^2}{5a^2} = \frac{a}{5}$ /

b) $I_x = \frac{2}{3}a^4 + \frac{3}{2}a^4 = \frac{13}{6}a^4 \quad I_y = \frac{8}{3}a^4 + \frac{14}{3}a^4 = \frac{22}{3}a^4 \quad I_{xy} = -a^4 - \frac{a^4}{2} = -\frac{3}{2}a^4$

c) $I_\xi = \frac{13}{6}a^4 - \left(\frac{a}{5}\right)^2 5a^2 = \frac{13}{6}a^4 - \frac{a^4}{5} = \frac{65 - 6}{30}a^4 = \frac{59}{30}a^4$

$I_\eta = \frac{22}{3}a^4 - 0 = \frac{22}{3}a^4 \quad I_{\xi\eta} = \frac{3}{2}a^4 - 0 = -\frac{3}{2}a^4$ /