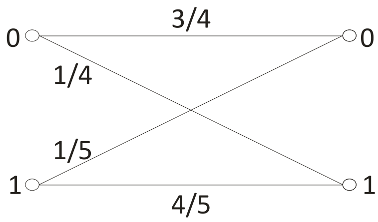
1. Neka je BC kanal predstavljen grafom



Odrediti sve vjerovatnoće (vjerovatnoću izlaznih simbola, uslovne vjerovatnoće i združene vjerovatnoće) ako je vjerovatnoća pojave jedinice na ulazu kanala jednaka $\frac{1}{4}$.

Rješenje:

Ulazni simboli su X_1 = 0 i X_2 = 1. Vjerovatnoća njihovog pojavljivanja na ulazu kanala je: $P(X_1) = 1 - P(X_2) = 1 - \frac{1}{4} = \frac{3}{4}$, $P(X_2) = \frac{1}{4}$

Simboli na izlazu iz kanala su: Y_1 = 0 i Y_2 = 1

Kanalna matrica je:

$$P = \begin{bmatrix} P(Y_1 \mid X_1) & P(Y_2 \mid X_1) \\ P(Y_1 \mid X_2) & P(Y_2 \mid X_2) \end{bmatrix}_{2\times 2} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{5} & \frac{4}{5} \end{bmatrix}_{2\times 2}$$

$$P(Y_1) = \sum_{i=1}^{2} P(X_i) \cdot P(Y_1 \mid X_i)$$

$$p(Y_1) = P(X_1) \cdot P(Y_1 \mid X_1) + P(X_2) \cdot P(Y_1 \mid X_2) = \frac{3}{4} \cdot \frac{3}{4} + \frac{1}{4} \cdot \frac{1}{5} = \frac{9}{16} + \frac{1}{20} = \frac{45+4}{80} = \frac{49}{80}$$

$$P(Y_2) = \sum_{i=1}^{2} P(X_i) \cdot P(Y_2 \mid X_i)$$

$$P(Y_2) = P(X_1) \cdot P(Y_2 \mid X_1) + P(X_2) \cdot P(Y_2 \mid X_2) = \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{4}{5} = \frac{3}{16} + \frac{4}{20} = \frac{15+16}{80} = \frac{31}{80}$$

Vjerovatnoće izlaznih simbola su $P(Y_1) = P(0) = \frac{49}{80}$ i $P(Y_2) = P(1) = \frac{31}{80}$

Zbir ovih vjerovatnoća mora biti jednak 1.

Aposteriorne vjerovatnoće su:

$$P(X_1 \mid Y_1) = \frac{P(X_1) \cdot p(Y_1 \mid X_1)}{P(Y_1)} = \frac{\frac{3}{4} \cdot \frac{3}{4}}{\frac{49}{80}} = \frac{\frac{9}{16}}{\frac{49}{80}} = \frac{45}{49}$$

$$P(X_2 \mid Y_2) = \frac{P(X_2) \cdot p(Y_2 \mid X_2)}{P(Y_2)} = \frac{\frac{1}{4} \cdot \frac{4}{5}}{\frac{31}{80}} = \frac{\frac{4}{20}}{\frac{31}{80}} = \frac{16}{31}$$

Iz uslova da je: $P(X_1 \mid Y_1) + P(X_2 \mid Y_1) = 1$ slijedi:

$$P(X_2 \mid Y_1) = 1 - P(X_1 \mid Y_1) = 1 - \frac{45}{49} = \frac{4}{49}$$

i $P(X_1 \mid Y_2) + P(X_2 \mid Y_2) = 1$ slijedi da je:

$$P(X_1 \mid Y_2) = 1 - P(X_2 \mid Y_2) = 1 - \frac{16}{31} = \frac{15}{31}$$

Združene vjerovatnoće su:

$$P(X_1, Y_1) = P(X_1) \cdot p(Y_1 \mid X_1) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$$

$$P(X_2, Y_2) = P(X_2) \cdot p(Y_2 \mid X_2) = \frac{1}{4} \cdot \frac{4}{5} = \frac{1}{5}$$

$$P(X_1, Y_2) = P(X_1) \cdot P(Y_2 \mid X_1) = \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{16}$$

$$P(X_2, Y_1) = P(X_2) \cdot P(Y_2 \mid X_1) = \frac{1}{4} \cdot \frac{1}{5} = \frac{1}{20}$$

Provjera: zbir ovih vjerovatnoća mora biti jednak jedinici.

2. Dat je pravougaoni kod (9, 4)

a) Izvršiti kodiranje poruke **1001** pomoću generišuće matrice. Bitove parnosti postaviti na poslednjim pozicijama u kodu.

b) Izvršiti dekodiranje poruke **110111110** pomoću kontrolne matrice

Rješenje:

a) Generišuća matrica pravougaonog koda (9, 4) ako se bitovi parnosti postavljaju na poslednjim pozicijama u kodu je dimenzija 4×9:

$$G = \left[\begin{array}{cccc:ccccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{array}\right]_{4\times 9}$$

i_1	i_2	c_1
i_3	i_4	c_2
c_3	c_4	c_5

c_1 - kontroliše i_1 i i_2

c_2 - kontroliše i_3 i i_4

c_3 - kontroliše i_1 i i_3

Paritetna matrica je:

$$P = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}_{4\times 5}$$

$$i \cdot G = C = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}_{1\times 4} \cdot \left[\begin{array}{cccc:ccccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{array}\right]_{4\times 9} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}_{1\times 9}$$

Kodirana poruka je **100111110**

b) $C \cdot H^T = S \qquad H = \left[P^T \;\vdots\; I_m \right]_{(n-k)\times n}$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}_{1\times 9} \cdot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{9\times 5} = \underline{\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \end{bmatrix}}_{1\times 5}$$

$$H = \begin{bmatrix} \boxed{1} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \boxed{0} & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ \boxed{1} & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ \boxed{0} & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ \boxed{1} & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{5\times 9}$$

$i_1 \quad i_2 \quad i_3 \quad i_4$

došlo je do greške pri prenosu na poziciji 2. (druga kolona kontrolne matrice)

Dekodirana riječ je: **1001**

3. Poruku **1001** kodirati Hemingovim kodom (7, 4).

Hemingov kod neka bude:

a) bitove parnosti dodati na poslednjim pozicijama

b) sa pravilnim binarnim rasporedom bitova

Rješenje:

a) $i \cdot G = C \qquad G = \left[I \;\vdots\; P \right]_{4\times 7} = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array} \right]$

c_1 – kontroliše bitove i_1, i_2 i i_4

c_2 – kontroliše bitove i_1, i_3 i i_4

c_3 – kontroliše bitove i_2, i_3 i i_4

$$C = \begin{bmatrix}1 & 0 & 0 & 1\end{bmatrix}_{1\times4} \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]_{4\times7} = \begin{bmatrix}1 & 0 & 0 & 1 & 0 & 0 & 1\end{bmatrix}_{1\times7}$$

b) $i \cdot G = C$

$$C = \begin{bmatrix}1 & 0 & 0 & 1\end{bmatrix}_{1\times4} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}_{4\times7} = \begin{bmatrix}0 & 0 & 1 & 1 & 0 & 0 & 1\end{bmatrix}_{1\times7}$$

4. Izvršiti dekodiranje poruke **1011001** pomoću kontrolne matrice Hemingovog koda (7, 4) sa pravilnim binarnim rasporedom bitova.

Rješenje:

$$C \cdot H^T = S$$

$$H = \left.\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}_{3\times7}\right\}\text{kontrolna matrica za dati kod}$$

$$\begin{bmatrix}1 & 0 & 1 & 1 & 0 & 0 & 1\end{bmatrix}_{1\times7} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{7\times3} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}_{1\times3}$$

Sindrom ukazuje na 1. kolonu kontrolne matrice, što znači da je došlo do greške na poziciji 1

Poslije ispravljanja greške imamo poruku

0011001

Kada se uklone kontrolni biti (1. 2. i 4. pozicija) imamo poruku:

1001

5. Vjerovatnoća pojave nule na ulazu kaskadne veze dva BC kanala je 0.35. Ako je vjerovatnoća uspješnog prenosa jedinice u kanalu 0.60, a vjerovatnoća uspješnog prenosa nule 0.75, odrediti:

a) Nacrtati graf kanala i napisati kanalnu matricu

b) Odrediti sve vjerovatnoće (vjerovatnoće izlaznih simbola, aposteriorne vjerovatnoće i združene vjerovatnoće)

c) Odrediti apriorne entropije ulazne i izlazne liste

Rješenje:

a) Vjerovatnoća uspješnog prenosa jedinice je 0.60 tj:

$P(1|1) = \mathrm{P}(\mathrm{Y}_2 | \mathrm{X}_2) = 0.60$, slijedi da je

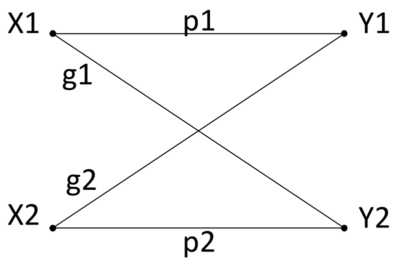
$P(0|1) = \mathrm{P}(\mathrm{Y}_1 | \mathrm{X}_2) = 1 - 0.60 = 0.40$.

Vjerovatnoća uspješnog prenosa nule je 0.75 tj:

$P(0|0) = \mathrm{P}(\mathrm{Y}_1 | \mathrm{X}_1) = 0.75$, slijedi da je

$P(1|0) = \mathrm{P}(\mathrm{Y}_2 | \mathrm{X}_1) = 1 - 0.75 = 0.25$.

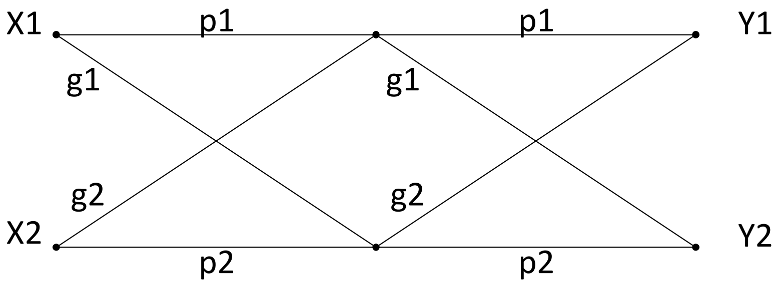
Graf jednog BC kanala je:



Kanalna matrica jednog BC kanala je:

$$P = \begin{bmatrix} P(\mathrm{Y}_1|\mathrm{X}_1) & P(\mathrm{Y}_2|\mathrm{X}_1) \\ P(\mathrm{Y}_1|\mathrm{X}_2) & P(\mathrm{Y}_2|\mathrm{X}_2) \end{bmatrix} = \begin{bmatrix} p_1 & g_1 \\ g_2 & p_2 \end{bmatrix}$$

Graf dva kaskadno vezana BC kanala je:



Kanalna matrica dva kaskadno vezana BC kanala je:

$$P=\begin{bmatrix} P(\mathrm{Y}_1\,|\,\mathrm{X}_1) & P(\mathrm{Y}_2\,|\,\mathrm{X}_1) \\ P(\mathrm{Y}_1\,|\,\mathrm{X}_2) & P(\mathrm{Y}_2\,|\,\mathrm{X}_2) \end{bmatrix}=\begin{bmatrix} p_1^2+g_1g_2 & p_1g_1+g_1p_2 \\ g_2p_1+p_2g_2 & p_2^2+g_2g_1 \end{bmatrix}=\begin{bmatrix} 0.6625 & 0.3375 \\ 0.54 & 0.46 \end{bmatrix}$$

Kako je zadatkom definisano da je

$P(\mathrm{X}_1)=\mathrm{P}(0)=0,35$, slijedi da je

$$P(\mathrm{X}_2)=\mathrm{P}(1)=1-P(\mathrm{X}_1)=1-0.35=0.65$$

b) Vjerovatnoće:

1. Vjerovatnoće izlaznih signala

$$P(\mathrm{Y}_1)=\mathrm{P}(\mathrm{X}_1)\,\mathrm{P}(\mathrm{Y}_1\,|\,\mathrm{X}_1)+\mathrm{P}(\mathrm{X}_2)\,\mathrm{P}(\mathrm{Y}_1\,|\,\mathrm{X}_2)$$

$$P(\mathrm{Y}_1)=0.35*0.6625+0.65*0.54=0.5828$$

$$P(\mathrm{Y}_2)=\mathrm{P}(\mathrm{X}_1)\,\mathrm{P}(\mathrm{Y}_2\,|\,\mathrm{X}_1)+\mathrm{P}(\mathrm{X}_2)\,\mathrm{P}(\mathrm{Y}_2\,|\,\mathrm{X}_2)$$

$$P(\mathrm{Y}_2)=0.35*0.3375+0.65*0.46=0.4171$$

2. Aposteriorne vjerovatnoće

$$P(\mathrm{X}_1\,|\,\mathrm{Y}_1)=\frac{P(\mathrm{X}_1)\,\mathrm{P}(\mathrm{Y}_1\,|\,\mathrm{X}_1)}{P(\mathrm{Y}_1)}=\frac{0.35*0.6625}{0.5828}=0.3978$$

$$P(\mathrm{X}_2\,|\,\mathrm{Y}_1)=1-\mathrm{P}(\mathrm{X}_1\,|\,\mathrm{Y}_1)=1-0.3978=0.6022$$

$$P(\mathrm{X}_2 \mid \mathrm{Y}_2) = \frac{P(\mathrm{X}_2)\,\mathrm{P}(\mathrm{Y}_2 \mid \mathrm{X}_2)}{P(\mathrm{Y}_2)} = \frac{0.65 * 0.46}{0.4171} = 0.7168$$

$$P(\mathrm{X}_1 \mid \mathrm{Y}_2) = 1 - \mathrm{P}(\mathrm{X}_2 \mid \mathrm{Y}_2) = 1 - 0.7168 = 0.2832$$

3. Združene vjerovatnoće

$$P(\mathrm{X}_1, \mathrm{Y}_1) = \mathrm{P}(\mathrm{X}_1)\,\mathrm{P}(\mathrm{Y}_1 \mid \mathrm{X}_1) = 0.35 * 0.6625 = 0.2318$$

$$P(\mathrm{X}_2, \mathrm{Y}_2) = \mathrm{P}(\mathrm{X}_2)\,\mathrm{P}(\mathrm{Y}_2 \mid \mathrm{X}_2) = 0.65 * 0.46 = 0.299$$

$$P(\mathrm{X}_1, \mathrm{Y}_2) = \mathrm{P}(\mathrm{X}_1)\,\mathrm{P}(\mathrm{Y}_2 \mid \mathrm{X}_1) = 0.35 * 0.3375 = 0.1181$$

$$P(\mathrm{X}_2, \mathrm{Y}_1) = \mathrm{P}(\mathrm{X}_2)\,\mathrm{P}(\mathrm{Y}_1 \mid \mathrm{X}_2) = 0.65 * 0.54 = 0.351$$

c) Apriorne entropije

Ulazna lista:

$$H(\mathrm{X}) = \sum_{i=1}^{r} P(\mathrm{X}_i) \log_2\left(\frac{1}{P(\mathrm{X}_i)}\right) = P(\mathrm{X}_1) \log_2 \frac{1}{P(\mathrm{X}_1)} + P(\mathrm{X}_2) \log_2 \frac{1}{P(\mathrm{X}_2)}$$

$$H(\mathrm{X}) = 0.35 * \log_2 \frac{1}{0.35} + 0.65 * \log_2 \frac{1}{0.65} = 0.9341$$

Izlazna lista:

$$H(\mathrm{Y}) = \sum_{j=1}^{s} P(\mathrm{Y}_j) \log_2\left(\frac{1}{P(\mathrm{Y}_j)}\right) = P(\mathrm{Y}_1) \log_2 \frac{1}{P(\mathrm{Y}_1)} + P(\mathrm{Y}_2) \log_2 \frac{1}{P(\mathrm{Y}_2)}$$

$$H(\mathrm{Y}) = 0.5828 * \log_2 \frac{1}{0.5828} + 0.4171 * \log_2 \frac{1}{0.4171} = 0.9801$$