i) $\vdash \forall x (A \to B) \to (\exists x A \to \exists x B)$

Rj1| Prvo dokažimo $\forall x (A \to B) \vdash \exists x A \to \exists x B$. Dokaz je niz formula:

$C_1 = \forall x (A \to B) \to (A \to B) \in T_\forall$, $C_2 = \forall x (A \to B) \in \Sigma$, $C_3 = A \to B$ (MP 2, 1),

$C_4 = B \to \exists x B \in T_\exists$, $C_5 = A \to \exists x B$ (TRANZ. 3, 4), $C_6 = \exists x A \to \exists x B$

($BP_\exists$ 5) (jer x nije slobodna u $\exists x B$).

Uočimo da u prethodnom dokazu koristimo jedno $BP\exists$ sa promenljivom x. Sa druge strane x nije slobodna u formuli $\forall x (A \to B)$, pa bez obzira što možda formula $\forall x (A \to B)$ nije rečenica mi možemo iskoristiti teoremu dedukcije. Tj. važi

$\vdash \forall x (A \to B) \to (\exists x A \to \exists x B)$

Rj2| Dokaz je niz formula: $C_1 = \forall x (A \to B) \to (A \to B) \in T_\forall$, $C_2 = B \to \exists x B \in T_\exists$,

$C_3 = (B \to \exists x B) \to (\forall x (A \to B) \to (B \to \exists x B)) \in T_1$, $C_4 = \forall x (A \to B) \to (B \to \exists x B)$ (MP 2, 3),

$C_5 = (A \to B) \to ((B \to \exists x B) \to (A \to \exists x B))$ (Teor. TRANZ.),

$C_6 = \forall x (A \to B) \to ((B \to \exists x B) \to (A \to \exists x B))$ (TRANZ. 1, 5),

$C_7 = C_6 \to (C_4 \to C_9) \in T_2$,

$C_8 = C_4 \to C_9$ (MP 6, 7),

$C_9 = \forall x (A \to B) \to (A \to \exists x B)$ (MP 4, 8),

$C_{10} = C_9 \to C_{11}$ (PERM. PP),

$C_{11} = A \to (\forall x (A \to B) \to \exists x B)$ (MP 9, 10),

$C_{12} = \exists x A \to (\forall x (A \to B) \to \exists x B)$ (BP$\exists$ 11) [x nije slobodna za $\forall x (A \to B) \to \exists x B$],

$C_{13} = C_{12} \to C_{14}$ (PERM. PP),

$C_{14} = \forall x (A \to B) \to (\exists x A \to \exists x B)$ (MP 12, 13)

Rj3| $C_1 = \forall x (A \to B) \to (A \to B) \in T_\forall$,

$C_2 = (A \to B) \to (\exists x A \to \exists x B)$ po pravilu $\dfrac{A \to B}{\exists x A \to \exists x B}$,

$C_3 = \forall x (A \to B) \to (\exists x A \to \exists x B)$ (TRANZ 1, 2).

ii) $\vdash ((\forall x A)\wedge(\forall x B)) \to \forall x (A\wedge B)$

Rj: Dokaz je sledeći niz formula:

$D_1 = (\forall x A)\wedge(\forall x B) \to \forall x A \in T_3,$

$D_2 = \forall x A \to A \in T_\forall,$

$D_3 = (\forall x A)\wedge(\forall x B) \to \forall x B \in T_4,$

$D_5 = \forall x B \to B \in T_\forall,$

$D_6 = (\forall x A)\wedge(\forall x B) \to A$ (TRANZ. 1,2),

$D_7 = (\forall x A)\wedge(\forall x B) \to B$ (TRANZ 3,4),

$D_8 = ((\forall x A)\wedge(\forall x B) \to A) \to (((\forall x A)\wedge(\forall x B) \to B) \to ((\forall x A)\wedge(\forall x B) \to A\wedge B))$ (Teor. inf.),

$D_9 = ((\forall x A)\wedge(\forall x B) \to B) \to ((\forall x A)\wedge(\forall x B) \to A\wedge B)$ (MP 6,8),

$D_{10} = (\forall x A)\wedge(\forall x B) \to (A\wedge B)$ (MP 7,9),

$D_{11} = (\forall x A)\wedge(\forall x B) \to \forall x (A\wedge B)$ (BP$\forall$ 10) (x nije slob. za $(\forall x A)\wedge(\forall x B)$).