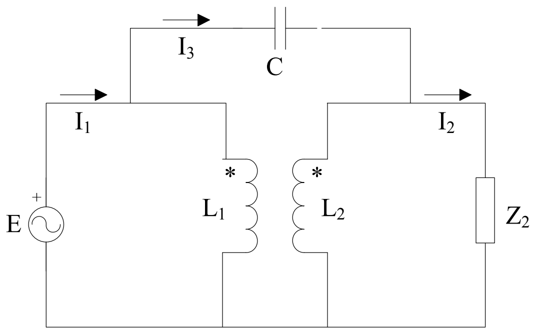
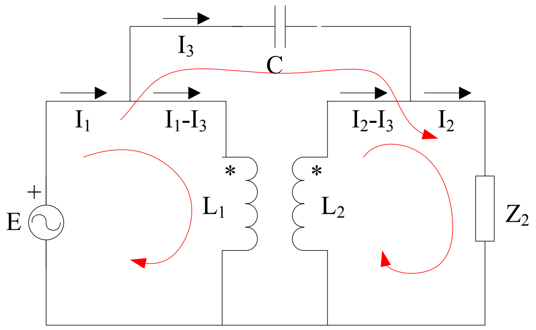
INDUKTIVNO SPREGNUTA KOLA

1. Data su spregnuta kola kao na slici.
a) Odrediti kapacitivnost kondenzatora C tako da pri kružnoj učestanosti ω struja I_2 bude nula.
b) Odrediti kompleksne struje $\underline{I}_1$, $\underline{I}_2$ i $\underline{I}_3$.
c) Da li se uslov pod a) može ostvariti ako se jedna od tačaka postavi na donji kraj kalema?



Rješenje:



a)

$$\underline{E} = j\omega L_1(\underline{I}_1 - \underline{I}_3) - j\omega L_{12}(\underline{I}_2 - \underline{I}_3) \qquad j\omega L_1\underline{I}_1 - j\omega L_{12}\underline{I}_2 + j\omega(L_{12} - L_1)\underline{I}_3 = \underline{E}$$

$$0 = Z_2\underline{I}_2 + j\omega L_2(\underline{I}_2 - \underline{I}_3) - j\omega L_{12}(\underline{I}_1 - \underline{I}_3) \quad => \quad -j\omega L_{12}\underline{I}_1 + (Z_2 + j\omega L_2)\underline{I}_2 + j\omega(L_{12} - L_2)\underline{I}_3 = 0$$

$$\underline{E} = \frac{1}{j\omega C}\underline{I}_3 + Z_2\underline{I}_2 \qquad Z_2\underline{I}_2 + \frac{1}{j\omega C}\underline{I}_3 = \underline{E}$$

$$\underline{I}_2 = \frac{D_2}{D} = 0 \Rightarrow D_2 = 0$$

$$D_2 = \begin{vmatrix} j\omega L_1 & \underline{E} & j\omega(L_{12} - L_1) \\ -j\omega L_{12} & 0 & j\omega(L_{12} - L_2) \\ 0 & \underline{E} & \dfrac{1}{j\omega C} \end{vmatrix} = 0$$

$$-\underline{E}j\omega(L_{12} - L_1)j\omega L_{12} - \underline{E}j\omega(L_{12} - L_2)j\omega L_1 + \underline{E}j\omega L_{12}\frac{1}{j\omega C} = 0$$

$$-\underline{E}\left(-\omega^2 L_{12}(L_{12} - L_1) - \omega^2 L_1(L_{12} - L_2) - \frac{L_{12}}{C}\right) = 0$$

$$\omega^2\left(L_{12}^2-\cancel{L_1L_{12}}+\cancel{L_1L_{12}}-L_1L_2\right)+\frac{L_{12}}{C}=0$$

$$\omega^2\left(L_1L_2-L_{12}^2\right)=\frac{L_{12}}{C}\Rightarrow C=\frac{L_{12}}{\omega^2\left(L_1L_2-L_{12}^2\right)}$$

Uz uslov da je $L_1L_2-L_{12}^2>0\Rightarrow L_{12}<\sqrt{L_1L_2}$

b) Na osnovu datih jednačina kola, i uslov da je I_2=0, jednostavno se dobija:

$$\underline{I}_3=j\omega C\underline{E}=j\frac{L_{12}}{\omega\left(L_1L_2-L_{12}^2\right)}\underline{E}$$

$$\underline{I}_2=0$$

$$\underline{I}_1=j\frac{L_{12}-L_2}{\omega\left(L_1L_2-L_{12}^2\right)}\underline{E}$$

c) Ako se jedna od tačaka postavi na donji kraj kalema tada $\underline{Z}_{12}$ mijenja znak, pa je uslov:

$$D_2=\begin{vmatrix} j\omega L_1 & \underline{E} & j\omega\left(-L_{12}-L_1\right) \\ j\omega L_{12} & 0 & j\omega\left(-L_{12}-L_2\right) \\ 0 & \underline{E} & \dfrac{1}{j\omega C}\end{vmatrix}=0\text{, tj.}$$

$$-\underline{E}j\omega\left(L_{12}+L_1\right)j\omega L_{12}+\underline{E}j\omega\left(L_{12}+L_2\right)j\omega L_1-\underline{E}j\omega L_{12}\frac{1}{j\omega C}=0$$

$$\omega^2L_{12}\left(L_{12}+L_1\right)-\omega^2L_1\left(L_{12}+L_2\right)-\frac{L_{12}}{C}=0$$

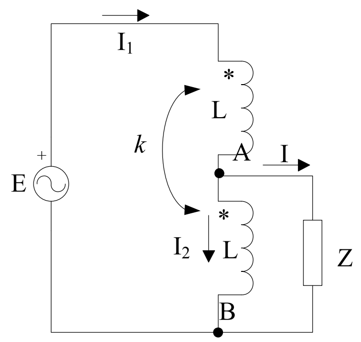
$$\omega^2\left(L_{12}^2+\cancel{L_1L_{12}}-\cancel{L_1L_{12}}-L_1L_2\right)-\frac{L_{12}}{C}=0$$

$$\omega^2\left(L_{12}^2-L_1L_2\right)=\frac{L_{12}}{C}\Rightarrow C=\frac{L_{12}}{\omega^2\left(L_{12}^2-L_1L_2\right)}$$

Uz uslov da je $L_{12}^2-L_1L_2>0\Rightarrow L_{12}>\sqrt{L_1L_2}$, koji je nemoguće ispuniti.

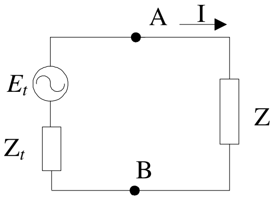
2. Dato je kolo prema šemi. Induktivnosti oba kalema su jednaka i iznose L. Koeficijent sprege između kalemova je k. Ako je EMS generatora $\underline{E}=E\angle 0$, odrediti kompleksnu struju u prijemniku impedanse $\underline{Z}$ pomoću Teveninove teoreme.

Brojni podaci: ωL=300Ω, k=0.9, $\underline{Z}$=R=20 Ω, E=100V.

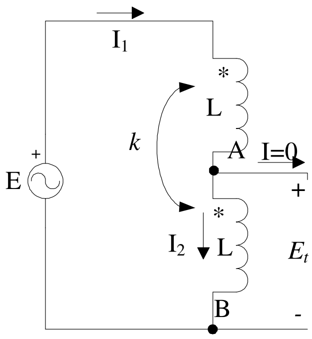


Rješenje:

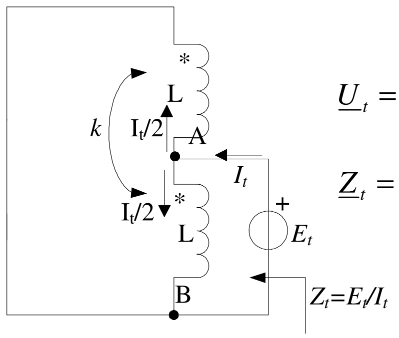
Na osnovu Teveninove teoreme, ekvivalentno kolo je:



$$\underline{I} = \frac{\underline{E}_t}{\underline{Z}+\underline{Z}_t} = \frac{\underline{E}_t}{R+\underline{Z}_t}$$, pri čemu se E_t dobija kada otkačimo A-B:



$\underline{I}_1 = \underline{I}_2$, a kako je kolo simetrično, važi da je $\underline{E}_t = \underline{E}/2$.



$$\underline{U}_t = j\omega L\frac{\underline{I}_t}{2} - j\omega kL\frac{\underline{I}_t}{2} = j\omega L(1-k)\frac{\underline{I}_t}{2}$$

$$\underline{Z}_t = \frac{\underline{U}_t}{\underline{I}_t} = \frac{j\omega L(1-k)}{2}$$

Konačno je, prema Teveninovoj teoremi:

$$\underline{I} = \frac{\underline{E}_t}{R+\underline{Z}_t} = \frac{\frac{1}{2}\underline{E}}{R+\frac{j\omega L(1-k)}{2}} = \frac{\underline{E}}{2R+j\omega L(1-k)}$$

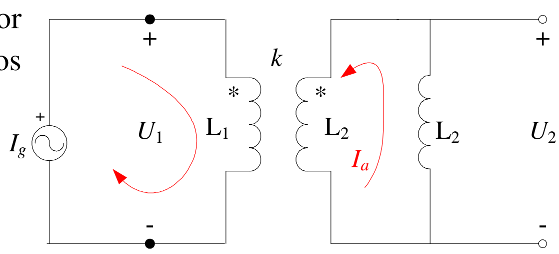
$$\underline{I} = \frac{100}{2\cdot 20 + j\cdot 300(1-0.9)}\frac{\text{V}}{\Omega} = \frac{100}{40+j30}\text{A} = \frac{10}{4+j3}\text{A}$$

$$\underline{I} = \frac{10}{4+j3}\frac{4-j3}{4-j3} = (1.6-j1.2)\,\text{A}$$

$$\underline{I} = 2e^{-j36.869^\circ}$$

3. Kolo sa slike je priključeno na strujni generator $i(t) = 5\sin\omega t$ A. Odrediti koeficijent sprege k tako da odnos efektivnih vrijednosti napona U_2/U_1 bude kn. U kojim granicama se može mijenjati n?

L_1=1 H, L_2=36 H.



Rješenje:

$L_{12} = k\sqrt{L_1 L_2} = k\sqrt{1\cdot 36} = 6k$ H

$\underline{U}_1 = j\omega L_1 \underline{I}_g + j\omega L_{12}\underline{I}_a$

$0 = j\omega L_2 \underline{I}_a + j\omega L_{12}\underline{I}_g + j\omega L_2 \underline{I}_a \Rightarrow L_{12}\underline{I}_g = -2L_2\underline{I}_a, \underline{I}_a = -\dfrac{L_{12}}{2L_2}\underline{I}_g$

$\Rightarrow \underline{U}_1 = j\omega\left(L_1 - \dfrac{L_{12}^2}{2L_2}\right)\underline{I}_g = j\omega L_1\left(1 - \dfrac{k^2}{2}\right)\underline{I}_g$

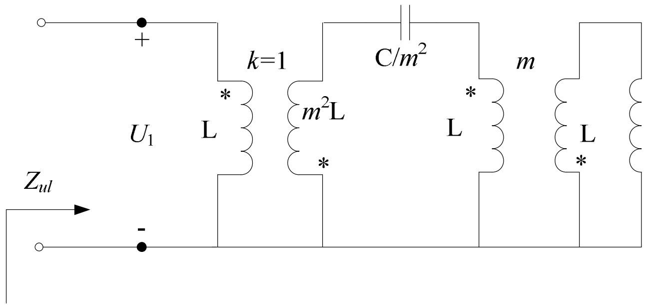
$\underline{U}_2 = -j\omega L_2 \underline{I}_a = \dfrac{j\omega L_{12}}{2}\underline{I}_g$

$$\frac{\underline{U}_2}{\underline{U}_1} = kn = \frac{\dfrac{\cancel{j\omega}L_{12}}{2}\cancel{\underline{I}_g}}{\cancel{j\omega}L_1\left(1-\dfrac{k^2}{2}\right)\cancel{\underline{I}_g}} = \frac{3k}{1\left(1-\dfrac{k^2}{2}\right)} = kn$$

$\Rightarrow 1 - \dfrac{k^2}{2} = \dfrac{3}{n}, k = \sqrt{2\left(1-\dfrac{3}{n}\right)}$

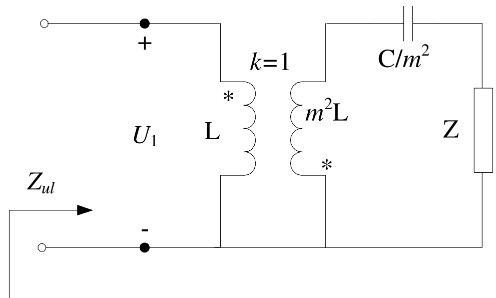
Za k=0, n=3; k=1, n=6.

4. Pokazati da ulazna impedansa kola prikazanog na slici ne zavisi od prenosnog odnosa m.



Rješenje:

Zadnji dio kola se može ekvivalentirati odgovarajućom impedansom, tako da se dobija kolo kao na slici:



$\dfrac{U_{prim2}}{U_{sek2}} = -m, \dfrac{I_{prim2}}{I_{sek2}} = -\dfrac{1}{m}$

$Z_{prim2} = Z = \dfrac{U_{prim2}}{I_{prim2}} = m^2 Z_{sek2} = m^2 j\omega L$

Sada je:

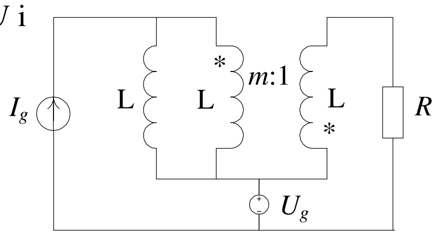
$\underline{U} = j\omega L\underline{I} + j\omega L_{12}\underline{I}_2,\ \ L_{12} = k\sqrt{Lm^2L} = mL$

$0 = \underline{I}_2\left(j\omega m^2 L + \dfrac{m^2}{j\omega C} + j\omega m^2 L\right) + j\omega L_{12}\underline{I} \Rightarrow \underline{I}_2 = -\dfrac{j\omega mL}{2j\omega m^2 L + \dfrac{m^2}{j\omega C}}\underline{I}$

$$\underline{U} = j\omega L\left(1 - \frac{j\omega m^2 L}{2j\omega m^2 L + \dfrac{m^2}{j\omega C}}\right)\underline{I}$$

$$\underline{Z}_{ul} = \frac{\underline{U}}{\underline{I}} = j\omega L\left(1 - \frac{j\omega m^2 L}{2j\omega m^2 L + \dfrac{m^2}{j\omega C}}\right) = \ldots = j\omega \frac{1-\omega^2 LC}{1-2\omega^2 LC} \neq f(m)$$

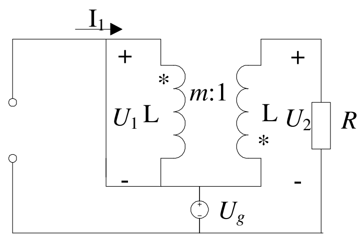
5. U kolu prema šemi djeluje naponski generator $u_g(t) = U$ i strujni generator struje $i_g(t) = \sqrt{2}I\cos\frac{R}{L}t$. Poznato je m, a parametri R, L i ω zadovoljavaju uslov $\omega = R/L$. Odrediti aktivnu snagu koju generatori ulažu u kolo.



Rješenje:

Metodom superpozicije je ukupna snaga jednaka zbiru snaga koje se dobijaju samo naponskim i samo strujnim generatorom.

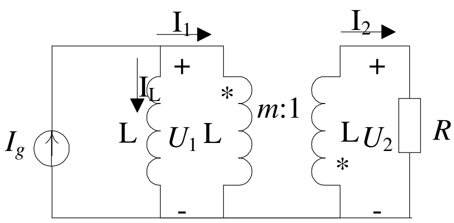
1) gasi se i_g (otvorena veza), a kako je u_g jednosmjeran napon, to je L=0 (kratak spoj):



$$I_1 = 0,\ U_1 = 0 = U_2 \Rightarrow U_R = u_g = U$$

$$P_{ugR} = \frac{U_R^2}{R} = \frac{U^2}{R}$$

2) gasi se u_g (kratak spoj):



$$\frac{U_1}{U_2} = -m, \frac{I_1}{I_2} = -\frac{1}{m}$$

$$\underline{U}_2 = R\underline{I}_2 = -mR\underline{I}_1;\ \ \underline{U}_1 = -m\underline{U}_2 = m^2 R\underline{I}_1$$

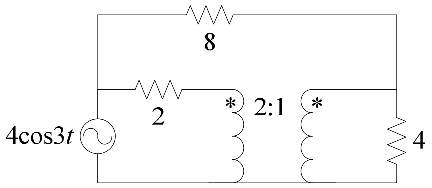
$$\underline{I}_g = \underline{I}_1 + \underline{I}_L = \frac{\underline{U}_1}{m^2 R} + \frac{\underline{U}_1}{j\omega L} = \underline{U}_1\left(\frac{1}{m^2 R} + \frac{1}{j\dfrac{R}{L}L}\right) = \frac{\underline{U}_1}{R}\left(\frac{1}{m^2} - j\right)$$

$$\Rightarrow \underline{U}_1 = \frac{R\underline{I}_g}{\left(\dfrac{1}{m^2} - j\right)} = \ldots = \left(\frac{m^2 R}{1+m^4} + j\frac{m^4}{1+m^4}\right)\underline{I}_g$$

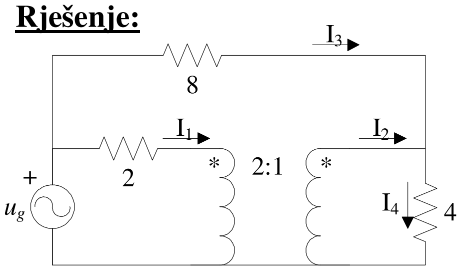
$$P_{ig} = \mathrm{Re}\{\underline{U}_1 \underline{I}_g\} = \frac{m^2}{1+m^4} R I_g^2$$

$$P = P_{ugR} + P_{ig}$$

6. Kolika je vrijednost srednje snage koja se utroši na otpornosti R=8?



Rješenje:



$$\underline{U}_g = 2\underline{I}_1 + \underline{U}_1 \Rightarrow \underline{U}_1 = 4 - 2\underline{I}_1$$

$$\frac{\underline{U}_1}{\underline{U}_2} = m = 2 \Rightarrow \underline{U}_2 = 2 - \underline{I}_1$$

$$\underline{I}_3 = \underline{I}_4 - \underline{I}_2 = \frac{\underline{U}_2}{4} - 2\underline{I}_1 = \frac{1}{2} - \frac{\underline{I}_1}{4} - 2\underline{I}_1 = \frac{1}{2} - \frac{9}{4}\underline{I}_1$$

$$\underline{U}_g = 8\underline{I}_3 + \underline{U}_2 = 8\left(\frac{1}{2} - \frac{9}{4}\underline{I}_1\right) + 2 - \underline{I}_1 = \ldots = 6 - 19\underline{I}_1$$

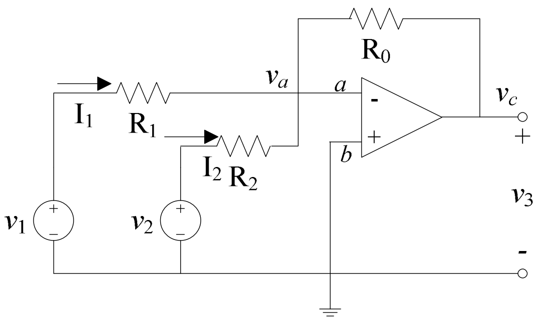
$$\Rightarrow \underline{I}_1 = \frac{6 - \underline{U}_g}{19} = \frac{2}{19}$$

$$\underline{I}_3 = \frac{1}{2} - \frac{9}{4}\frac{2}{19} = \frac{5}{19}$$

$$P = RI_3^2 = 0.554\text{W}$$

KOLA SA OPERACIONIM POJAČAVAČEM

1. Pokazati da je $v_3 = -R_0\left(\frac{v_1}{R_1} + \frac{v_2}{R_2}\right)$.



Rješenje:

$$v_a = v_b = 0; \quad v_c = v_3$$

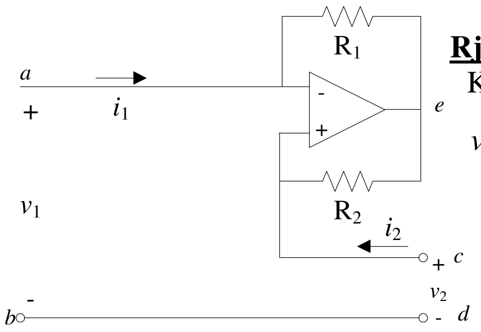
$$\frac{v_1 - v_a}{R_1} + \frac{v_2 - v_a}{R_2} = \underline{I}_3$$

$$v_3 = v_a - \underline{I}_3 R_0 = -R_0\left(\frac{v_1}{R_1} + \frac{v_2}{R_2}\right)$$

II način:

$$v_a\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-\frac{v_c}{R_0}=\frac{v_1}{R_1}+\frac{v_2}{R_2},v_a=0\Rightarrow v_3=-R_0\left(\frac{v_1}{R_1}+\frac{v_2}{R_2}\right)$$

2. Pokazati da je $v_1=v_2$, i $i_1=\frac{R_2}{R_1}i_2$.

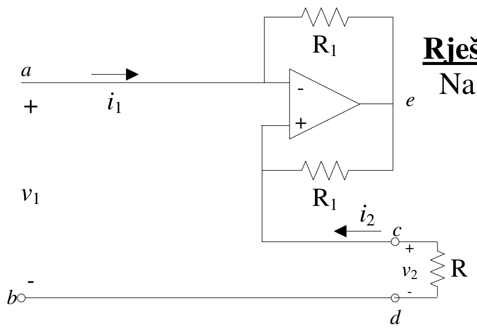


Rješenje:

Kako je riječ o idealnom OP, lako se zaključuje da je $v_1=v_2$.

$$v_1=i_1R_1+v_e=v_2=i_2R_2+v_e\Rightarrow i_1=\frac{R_2}{R_1}i_2$$

3. Za kolo na slici odrediti ulaznu otpornost R_{ab}.



Rješenje:

Na osnovu prethodnog zadatka je:

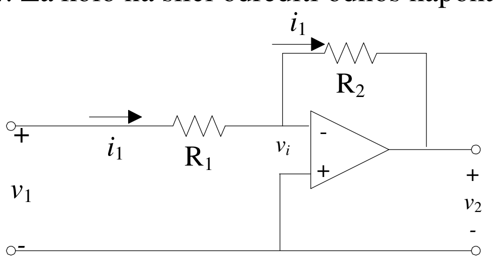
$$i_1=\frac{R_1}{R_1}i_2=i_2,\quad v_1=v_2$$

$$v_2=-Ri_2=-Ri_1=v_1$$

$$R_{ab}=\frac{v_1}{i_1}=\frac{-Ri_1}{i_1}=-R$$

Tj. kolo vrši konverziju otpornosti u negativnu otpornost.

4. Za kolo na slici odrediti odnos napona v_2/v_1 ako OP ima pojačanje A.



Rješenje:

$$v_i\neq 0$$

$$v_2=Av_i$$

$$-v_1+R_1i_1-v_i=0\Rightarrow -v_1+R_1i_1-\frac{v_2}{A}=0$$

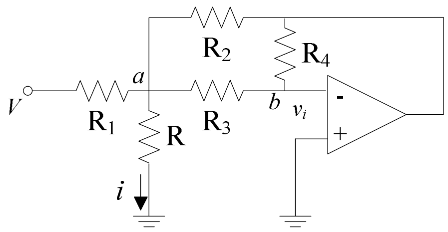
$$-v_1+\left(R_1+R_2\right)i_1+v_2=0\Rightarrow i_1=\frac{v_1-v_2}{R_1+R_2}$$

Kombinujući poslednje dvije relacije dobija se:

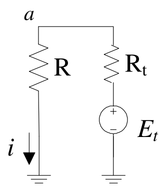
$$-v_1+\frac{R_1}{R_1+R_2}\left(v_1-v_2\right)-\frac{v_2}{A}=0\Rightarrow\frac{v_2}{v_1}=-\frac{A\frac{R_2}{R_1}}{1+\frac{R_2}{R_1}+A}$$

Za $A\to\infty\Rightarrow\frac{v_2}{v_1}\to-\frac{R_2}{R_1}$

5. Primjenom Teveninove teoreme naći vrijednost struje i u kolu sa slike.
Brojni podaci: V=3V, R=2kΩ, R_1=1kΩ, R_2=2kΩ, R_3=4kΩ, R_4=6kΩ.

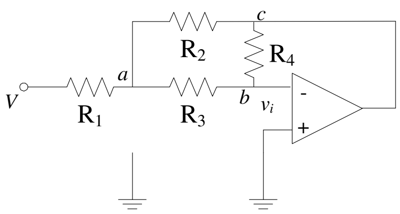


Rješenje:
Ekvivalentno kolo bi bilo kao na slici:



$$i = \frac{E_t}{R + R_t}$$

Određivanje E_t:



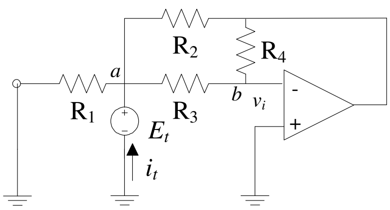
$$v_a\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-\frac{v_b}{R_3}-\frac{v_c}{R_2}=\frac{V}{R_1}$$

$$-\frac{v_a}{R_3}+v_b\left(\frac{1}{R_3}+\frac{1}{R_4}\right)-\frac{v_c}{R_4}=0$$

$$v_b = 0$$

Na osnovu gornjih relacija dobija se: $v_a = \frac{6}{5}V = E_t$

Određivanje R_t:



$$R_t = \frac{v_a\left(=E_t\right)}{i_t}$$

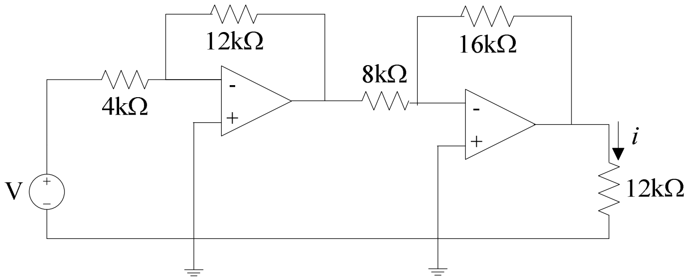
$$v_b\left(\frac{1}{R_3}+\frac{1}{R_4}\right)-\frac{v_a}{R_3}-\frac{v_c}{R_4}=0,\quad v_b=0 \Rightarrow v_c = -\frac{R_4}{R_3}v_a$$

$$i_t = \frac{v_a}{R_1}+\frac{v_a - v_b}{R_3}+\frac{v_a - v_c}{R_2} = v_a\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{R_4}{R_2R_3}\right)$$

Odakle je R_t=400Ω.

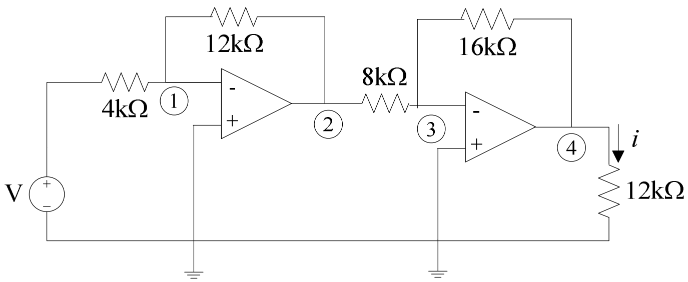
Konačno je $i = \frac{1.2V}{\left(2000+400\right)\Omega} = 0.5mA$

6. Dato je kolo prema šemi. Odrediti struju *i*. V=2V.



Rješenje:

Označimo čvorove na sledeći način:



Važi set jednačina:

Za čvor 1:

$$\left(\frac{1}{4}+\frac{1}{12}\right)v_1-\frac{1}{12}v_2=\frac{V}{4},\quad v_1=0\Rightarrow v_2=-3V$$

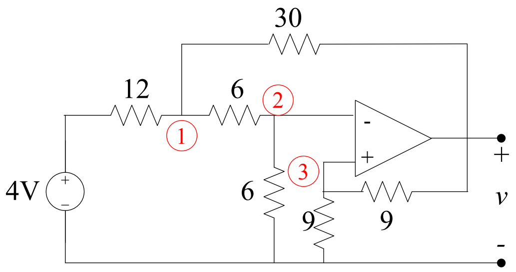
Za čvor 3:

$$-\frac{1}{8}v_2+\left(\frac{1}{8}+\frac{1}{16}\right)v_3-\frac{1}{16}v_4=0,\quad v_3=0\Rightarrow v_4=-2v_2=6V$$

Konačno je:

$$i=\frac{v_4}{12\mathrm{k}\Omega}=\frac{6V}{12\mathrm{k}\Omega}=\frac{12\mathrm{V}}{12\mathrm{k}\Omega}=1\mathrm{mA}$$

7. Za kolo sa slike naći napon v.



Rješenje:

Za označene čvorove važe sljedeće jednačine:

$$\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{30}\right)v_1-\frac{1}{6}v_2-\frac{1}{30}v=\frac{v_g}{12}$$

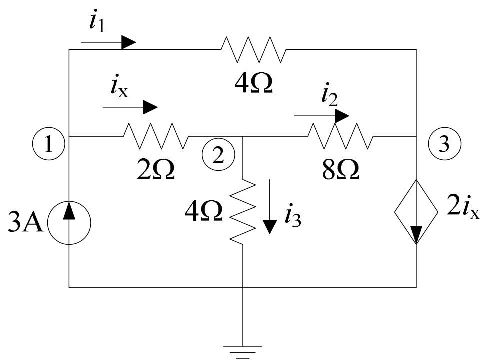
$$-\frac{1}{6}v_1+\left(\frac{1}{6}+\frac{1}{6}\right)v_2=0\Rightarrow v_1=2v_2$$

$$\left(\frac{1}{9}+\frac{1}{9}\right)v_3-\frac{1}{9}v=0$$

, a zbog idealnosti operacionog pojačavača važi i da je $v_3=v_2$. Odatle je $v=v_g/2=2\mathrm{V}$.

KONTROLISANI IZVORI

1. Metodom potencijala čvorova naći sve struje u kolu.



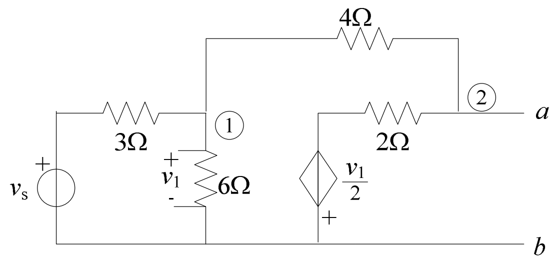
Rješenje:
Jednačine za sve čvorove su:

$$\left(\frac{1}{4}+\frac{1}{2}\right)v_1-\frac{1}{2}v_2-\frac{1}{4}v_3=3A$$
$$-\frac{1}{2}v_1+\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)v_2-\frac{1}{8}v_3=0$$
$$-\frac{1}{4}v_1-\frac{1}{8}v_2+\left(\frac{1}{4}+\frac{1}{8}\right)v_3=-2i_x$$
$$\frac{v_1-v_2}{2}=i_x$$

Odakle se dobijaju sledeće vrijednosti:

$$v_1=4.8V,\ v_2=2.4V,\ v_3=-2.4V$$
$$i_1=\frac{v_1-v_3}{4}=\frac{7.2}{4}A=1.8A$$
$$i_2=\frac{v_2-v_3}{8}=\frac{4.8}{8}A=0.6A$$
$$i_3=\frac{v_2}{4}=\frac{2.4}{4}A=0.6A$$
$$i_x=\frac{v_1-v_2}{2}=\frac{2.4}{2}A=1.2A$$

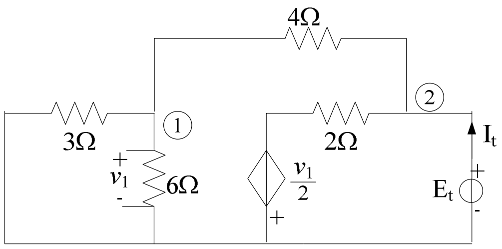
2. Odrediti ekvivalentni Teveninov generator u odnosu na tačke *a*-*b*.



Rješenje:
Sa slike se vidi da je $E_t=v_2$. Set jednačina za određivanje napona v_2 je:

$$\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{6}\right)v_1-\frac{1}{4}v_2=\frac{v_s}{3}$$
$$-\frac{1}{4}v_1+\left(\frac{1}{4}+\frac{1}{2}\right)v_2=-\frac{v_1/2}{2}$$

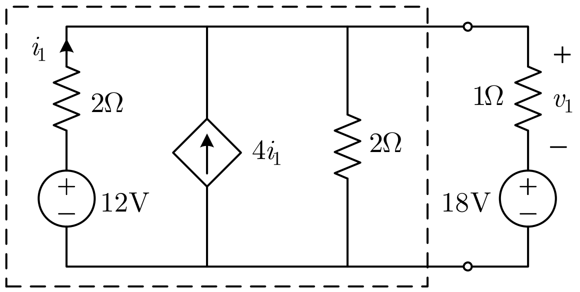
Odakle se dobija da je $v_2=E_t=0$.
Ekvivalentno kolo za određivanje R_t je kao na slici:



$$\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{4}\right)v_1-\frac{1}{4}v_2=0$$
$$-\frac{1}{4}v_1+\left(\frac{1}{4}+\frac{1}{2}\right)v_2=i_t-\frac{v_1/2}{2}$$

Odakle je $i_t=\frac{3}{4}v_2$, a $R_t=\frac{v_2}{i_t}=\frac{4}{3}\Omega$.

3. Zamijeniti označeni dio kola sa ekvivalentnim Teveninovim generatorom. Odrediti napon v_1.

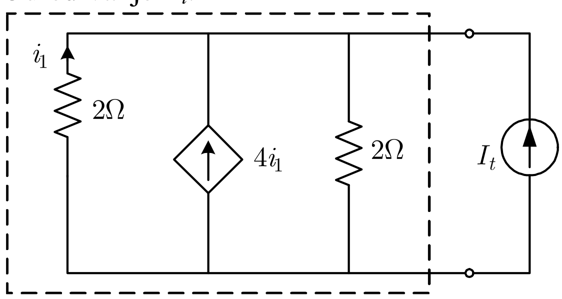


Rješenje:
Određivanje E_t:

$$(i_1+4i_1)\cdot R_{=2\Omega}=E_t$$
$$V_{=12\mathrm{V}}=2i_1+E_t$$

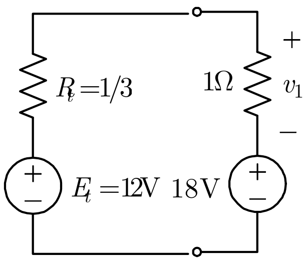
Na osnovu čega slijedi da je i_1=1A, a E_t=10V.

Određivanje R_t:



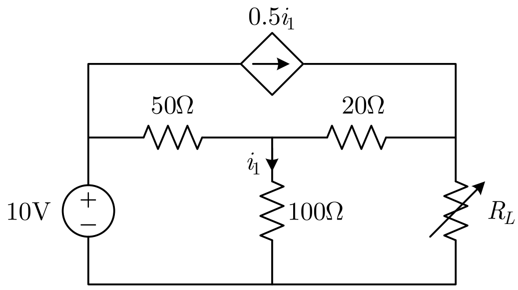
$$R_t=\frac{E_t}{I_t}$$
$$E_t=(5i_1-I_t)\cdot R$$
$$E_t=-Ri_1=-2i_1$$

Odavde se dobija da je I_t=-6i_1, pa je R_t=1/3, pa je ekvivalentno kolo:



$$I\left(1+\frac{1}{3}\right)+18=10 \Rightarrow I=-6A$$
$$v_1 = I\cdot 1 = -6V$$

4. Opterećenje R_L se podešava u kolu sve dok se ne postigne maksimalna snaga u njemu.
a) Odrediti vrijednost za R_L.
b) Kolika je ta maksimalna snaga?

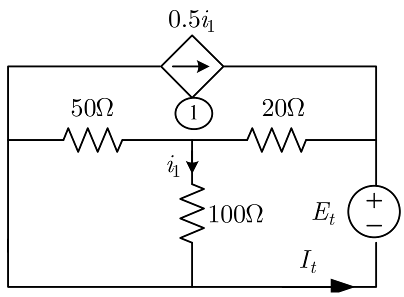


Rješenje:

Koristeći Teveninovu teoremu na mjestu R_L, određuje se E_t:

$$E_t = 0.5i_1\cdot 20 + i_1\cdot 100 = 110i_1$$
$$10\text{V}=\left(i_1-0.5i_1\right)\cdot 50+100i_1=125i_1 \Rightarrow i_1=0.08\text{A}$$
$$\Rightarrow E_t = 8.8\text{V}$$

Da bi se odredilo R_t, ekvivalentno kolo je:



$$\left(\frac{1}{50}+\frac{1}{20}+\frac{1}{100}\right)v_1-\frac{1}{20}E_t=0 \Rightarrow E_t=1.6v_1=160i_1$$

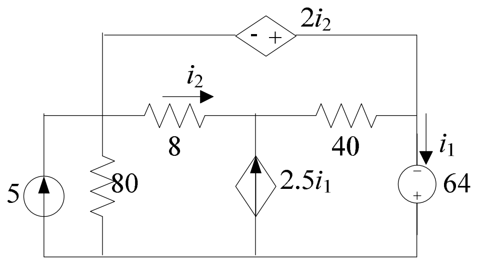
S druge strane je:

$$E_t=100i_1+20\left(I_t+0.5i_1\right) \Rightarrow 160i_1=110i_1+20I_t$$
$$I_t=2.5i_1$$
$$R_t=\frac{160i_1}{2.5i_1}=64\Omega=R_L$$

Konačno je snaga:

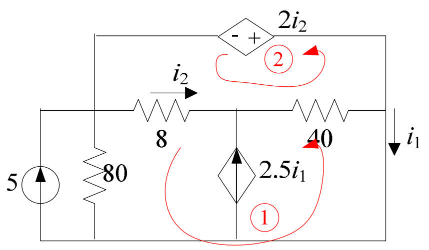
$$P_{\max} = \frac{E_t^2}{4R_L} = \frac{8.8^2}{4 \cdot 64} = 0.3025\mathrm{W}$$

5. Dato je kolo kao na slici. Metodom superpozicije naći vrijednosti struja i_1 i i_2.



Rješenje:

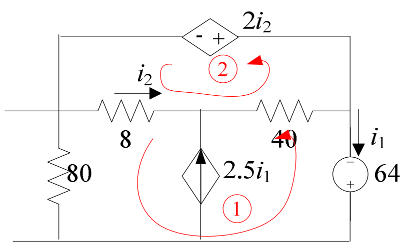
Kada djeluje samo strujni generator, dobija se set jednačina za konture označene na slici:



$$80(5+2.5i_1 - i_1) - 40(i_2 + 2.5i_1) - 8i_2 = 0$$
$$8i_2 + 40(i_2 + 2.5i_1) + 2i_2 = 0$$

Odatle se dobija da je $i_1 = -\frac{100}{29}$, $i_2 = \frac{200}{29}$

Sada djeluje samo naponski generator:



$$8i_2 + 40(2.5i_1 + i_2) - 64 + 80(i_1 - 2.5i_1) = 0$$
$$8i_2 + 40(2.5i_1 + i_2) + 2i_2 = 0$$

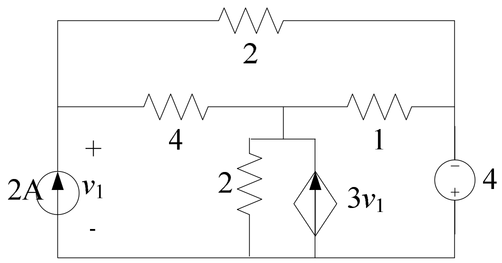
Odakle se dobija da je $i_1 = -\frac{16}{29}$, $i_2 = \frac{32}{29}$.

Superpozicijom se ove dvije vrijednosti sabiraju, pa je konačno:

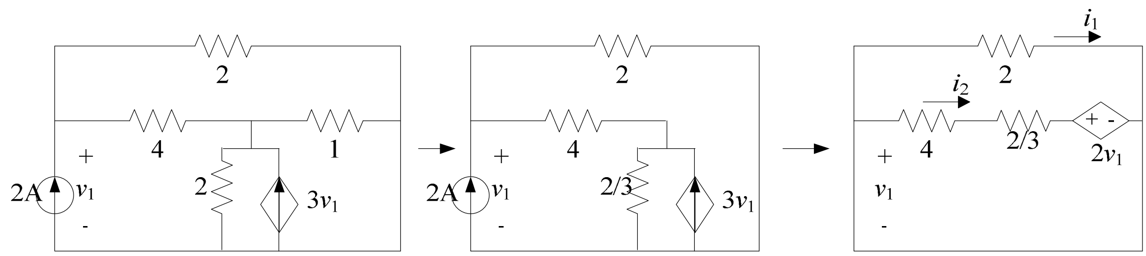
$$i_1 = -\frac{100}{29} - \frac{16}{29} = -4\text{A}$$

$$i_2 = \frac{200}{29} + \frac{32}{29} = 8\text{A}$$

6. Dato je kolo kao na slici. Metodom superpozicije naći snagu koja se troši na otpornosti od 4Ω.



Rješenje:
Samo strujni generator:



Sa poslednje slike važi:

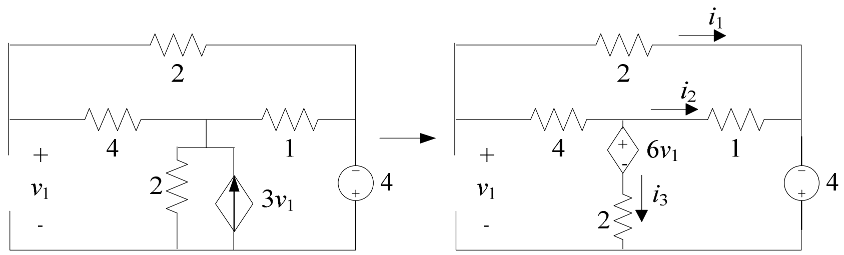
$$i_1 + i_2 = 2$$

$$2i_1 - 2v_1 - \left(\frac{2}{3} + 4\right) i_2 = 0$$

$$v_1 = 2i_1$$

Pa je $i_1 = \frac{7}{2},\ i_2 = -\frac{3}{2}$.

Samo naponski generator:



$$(2+4)i_1 - 1i_2 = 0$$

$$1i_2 - 4 - 2i_3 - 6v_1 = 0$$

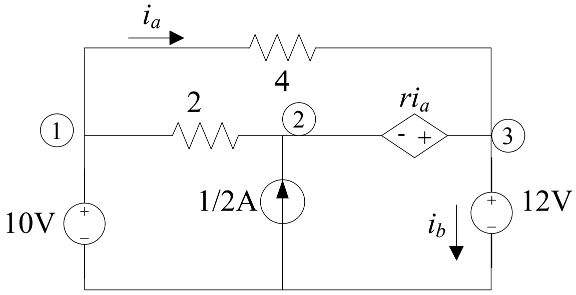
$$i_1 + i_2 + i_3 = 0$$

$$v_1 = 2i_1 - 4$$

Odakle je struja kroz otpornik 4Ω i_1=-2.5.

Superpozicijom je ukupna struja kroz željenu otpornost i=-1.5-2.5=-4A, dok je snaga $P=Ri^2$=64W.

7. Potencijali čvorova u kolu sa slike su $v_1 = 10\text{V}$, $v_2 = 14\text{V}$ i $v_3 = 12\text{V}$. Naći struju i_b i konstantu r.



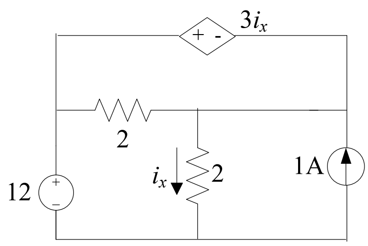
Rješenje:
Od čvorova 1 i 2 može se formirati jedan superčvor, i primjenom KCL na njega dobija se:

$$\frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2} + \frac{1}{2} - i_b = 0 \Rightarrow i_b = -2\text{A}$$

$$i_a = \frac{v_1 - v_3}{4} = -\frac{1}{2}$$

$$ri_a = v_3 - v_2 \Rightarrow r = 4\frac{\text{V}}{\text{A}}$$

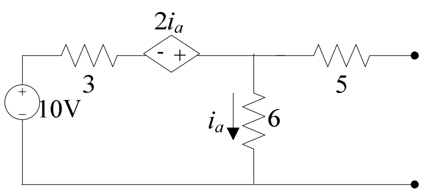
8. Odrediti struju i_x u kolu sa slike.



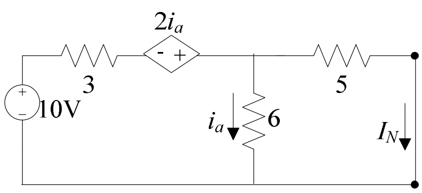
Rješenje:

$$3i_x = 12 - 2i_x \Rightarrow i_x = 2.4\text{A}$$

9. Za kolo sa slike naći parametre Nortonovog generatora.



Rješenje:
Određivanje I_N:

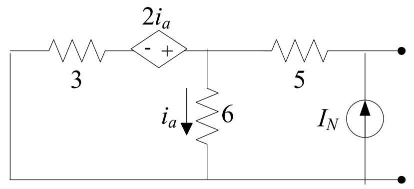


$$10 = 3(i_a + I_N) - 2i_a + 6i_a \Rightarrow 7i_a + 3I_N = 10$$

$$5I_N - 6i_a = 0$$

Odakle se dobija da je I_N=1.13A.

Određivanje R_N:

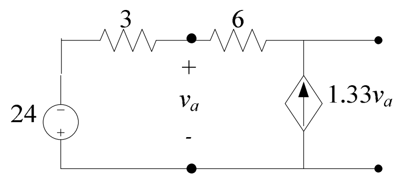


$$3(i_a - I_N) - 2i_a + 6i_a = 0 \Rightarrow i_a = \frac{3}{7} I_N$$

$$E_N = 5I_N + 6i_a = \frac{53}{7} I_N$$

$$G_N = \frac{I_N}{E_N} = \frac{7}{53} = 0.132 \text{ S}$$

10. Za kolo sa slike naći Nortonov ekvivalent.



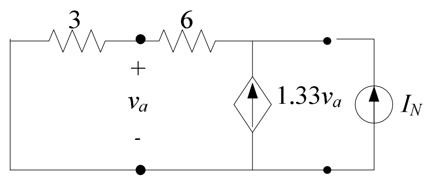
Rješenje:

Određivanje I_N znači da kratko spojimo krajeve kola.

$$i(3+6) + 24 = 0 \Rightarrow i = -\frac{8}{3} = \frac{v_a}{6} \Rightarrow v_a = -16\text{V}$$

$$I_N = i + 1.33v_a = -24\text{A}$$

Određivanje G_N:



$$I_N + 1.33v_a = \frac{E_N}{6+3}$$

$$v_a = -6(1.33v_a + I_N) + E_N$$

Odakle je E_N=3v_a, pa je:

$$I_N = \frac{3v_a}{9} - 1.33v_a = -v_a$$

$$G_N = \frac{I_N}{E_N} = -\frac{1}{3}\text{S}$$