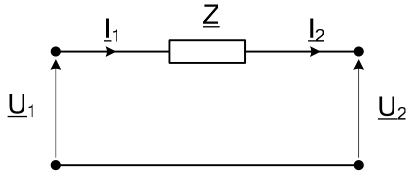
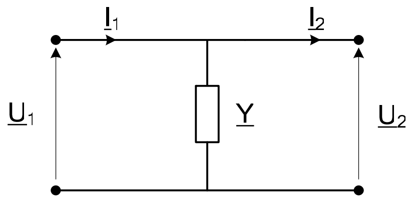
1. Odrediti „a“ parametre sljedećih mreža sa dva para krajeva:

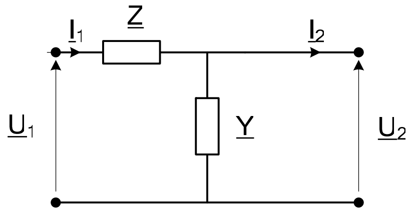
a)



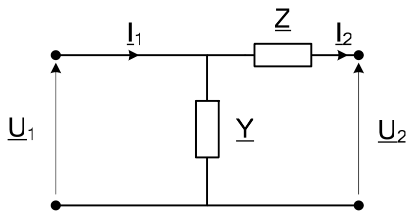
b)



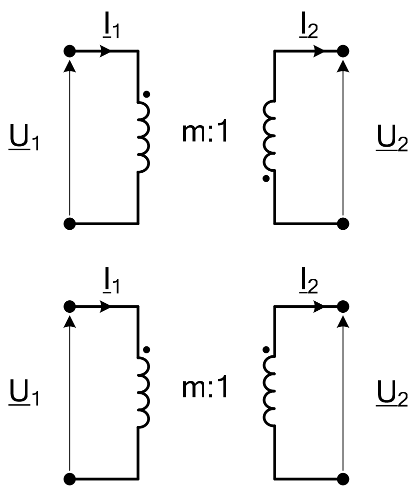
c)



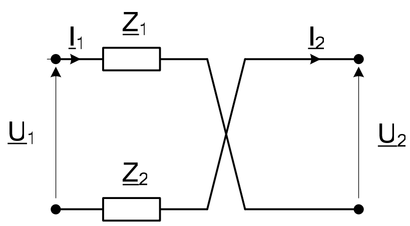
d)



e)



f)



<u>**R**</u>

a)

$$\left.\begin{array}{l}\underline{U}_1=\underline{U}_2+\underline{Z}\underline{I}_2\\ \underline{I}_1=\qquad\quad \underline{I}_2\end{array}\right\}\rightarrow\begin{bmatrix}\underline{U}_1\\ \underline{I}_1\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}\underline{U}_2\\ \underline{I}_2\end{bmatrix}\rightarrow\begin{array}{ll}a_{11}=1 & a_{12}=\underline{Z}\\ a_{21}=0 & a_{22}=1\end{array}$$

b)

$$\left.\begin{array}{l}\underline{U}_1=\underline{U}_2\\ \underline{I}_1=\underline{Y}\underline{U}_2+\underline{I}_2\end{array}\right\}\rightarrow\begin{bmatrix}\underline{U}_1\\ \underline{I}_1\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}\underline{U}_2\\ \underline{I}_2\end{bmatrix}\rightarrow\begin{array}{ll}a_{11}=1 & a_{12}=0\\ a_{21}=\underline{Y} & a_{22}=1\end{array}$$

c)

$$\left.\begin{array}{l}\underline{U}_1=\underline{Z}\underline{I}_1+\underline{U}_2\\ \underline{I}_1=\underline{Y}\underline{U}_2+\underline{I}_2\end{array}\right\}\rightarrow\left.\begin{array}{l}\underline{U}_1=\underline{Z}\left(\underline{Y}\underline{U}_2+\underline{I}_2\right)+\underline{U}_2\\ \underline{I}_1=\underline{Y}\underline{U}_2+\underline{I}_2\end{array}\right\}\rightarrow$$

$$\left.\begin{array}{l}\underline{U}_1=\left(\underline{Z}\underline{Y}+1\right)\underline{U}_2+\underline{Z}\underline{I}_2\\ \underline{I}_1=\underline{Y}\underline{U}_2+\underline{I}_2\end{array}\right\}\rightarrow\begin{array}{ll}a_{11}=\left(\underline{Z}\underline{Y}+1\right) & a_{12}=\underline{Z}\\ a_{21}=\underline{Y} & a_{22}=1\end{array}$$

d)

$$\left.\begin{array}{l}\underline{U}_1=\underline{Z}\underline{I}_2+\underline{U}_2\\ \underline{I}_1=\underline{Y}\left(\underline{Z}\underline{I}_2+\underline{U}_2\right)+\underline{I}_2\end{array}\right\}\rightarrow\left.\begin{array}{l}\underline{U}_1=\underline{U}_2+\underline{Z}\underline{I}_2\\ \underline{I}_1=\underline{Y}\underline{U}_2+\left(\underline{Y}\underline{Z}+1\right)\underline{I}_2\end{array}\right\}\rightarrow\begin{array}{ll}a_{11}=1 & a_{12}=\underline{Z}\\ a_{21}=\underline{Y} & a_{22}=1+\underline{Y}\underline{Z}\end{array}$$

e)

IT sa saglasnim krajevima

$$\left.\begin{array}{l}\underline{U}_1=-m\underline{U}_2\\ \underline{I}_1=-\dfrac{1}{m}\underline{I}_2\end{array}\right\}\rightarrow\begin{array}{ll}a_{11}=-m & a_{12}=0\\ a_{21}=0 & a_{22}=-\dfrac{1}{m}\end{array}$$

IT sa nesaglasnim krajevima

$$\left.\begin{array}{l}\underline{U}_1=m\underline{U}_2\\ \underline{I}_1=\dfrac{1}{m}\underline{I}_2\end{array}\right\}\rightarrow\begin{array}{ll}a_{11}=m & a_{12}=0\\ a_{21}=0 & a_{22}=\dfrac{1}{m}\end{array}$$

f)

$$\left.\begin{array}{l}\underline{U}_1=\underline{Z}_1\underline{I}_1-\underline{U}_2-\underline{Z}_2\underline{I}_2\\ \underline{I}_1=-\underline{I}_2\end{array}\right\}\rightarrow\left.\begin{array}{l}\underline{U}_1=-\underline{U}_2-\left(\underline{Z}_1+\underline{Z}_2\right)\underline{I}_2\\ \underline{I}_1=-\underline{I}_2\end{array}\right\}\rightarrow\begin{array}{ll}a_{11}=-1 & a_{12}=-\left(\underline{Z}_1+\underline{Z}_2\right)\\ a_{21}=0 & a_{22}=-1\end{array}$$

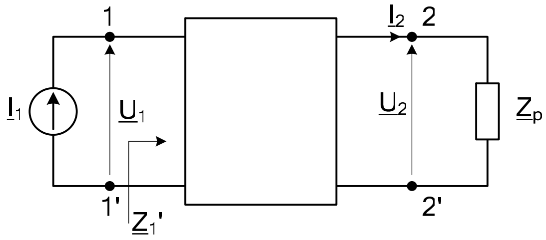
2. Posmatra se mreža prema šemi. Da li je impedansa kola data izrazom $\underline{Z}_1^{'}=\dfrac{\underline{Z}_{10}\underline{Z}_p+\underline{Z}_{1k}\underline{Z}_T}{\underline{Z}_p+\underline{Z}_T}$, gdje su:

$\underline{Z}_{10}$ - ulazna impedansa mreže sa strane 1-1' kada su krajevi 2-2' otvoreni

$\underline{Z}_{1k}$ - ulazna impedansa mreže sa strane 1-1' kada su krajevi 2-2' zatvoreni

$\underline{Z}_T$ - impedansa Teveninovog generatora u odnosu na krajeve 2-2'

$\underline{Z}_p$ - impedansa prijemnika



R

Ulazna impedansa na krajevima 1-1'

$$\underline{Z}_1' = \frac{\underline{U}_1}{\underline{I}_1}$$

Ako se iskoriste „a" parametri

$$\begin{aligned} \underline{U}_1 &= \underline{a}_{11}\underline{U}_2 + \underline{a}_{12}\underline{I}_2 \\ \underline{I}_1 &= \underline{a}_{21}\underline{U}_2 + \underline{a}_{22}\underline{I}_2 \end{aligned} \quad (\#)$$

A ako se još doda relacija koja definiše uslove na krajevima 2-2'

$$\underline{U}_2 = \underline{Z}_p\underline{I}_2$$

Kako se traži ulazna impedansa podijele se jednačine iz sistema (#)

$$\underline{Z}_1' = \frac{\underline{U}_1}{\underline{I}_1} = \frac{\underline{a}_{11}\underline{U}_2 + \underline{a}_{12}\underline{I}_2}{\underline{a}_{21}\underline{U}_2 + \underline{a}_{22}\underline{I}_2} = \frac{\underline{a}_{11}\underline{Z}_p\underline{I}_2 + \underline{a}_{12}\underline{I}_2}{\underline{a}_{21}\underline{Z}_p\underline{I}_2 + \underline{a}_{22}\underline{I}_2} = \frac{\underline{a}_{11}\underline{Z}_p + \underline{a}_{12}}{\underline{a}_{21}\underline{Z}_p + \underline{a}_{22}}$$

Ako se izraz podijeli sa $\underline{a}_{21}$

$$\underline{Z}_1' = \frac{\frac{\underline{a}_{11}}{\underline{a}_{21}}\underline{Z}_p + \frac{\underline{a}_{12}}{\underline{a}_{21}}}{\underline{Z}_p + \frac{\underline{a}_{22}}{\underline{a}_{21}}}$$

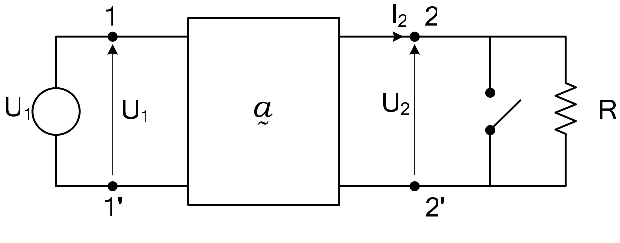
Ako se impedanse definisane u zadatku izraze preko „a" parametara

$$\underline{Z}_{10} = \left.\frac{\underline{U}_1}{\underline{I}_1}\right|_{\underline{I}_2=0} = \frac{\underline{a}_{11}}{\underline{a}_{21}} \qquad \underline{Z}_{1k} = \left.\frac{\underline{U}_1}{\underline{I}_1}\right|_{\underline{U}_2=0} = \frac{\underline{a}_{12}}{\underline{a}_{22}} \qquad \underline{Z}_T = \left.\frac{\underline{U}_2}{-\underline{I}_2}\right|_{\underline{I}_1=0} = \frac{\underline{a}_{22}}{\underline{a}_{21}}$$

Zamjenom u prethodnu relaciju, dolazi se do

$$\underline{Z}_1' = \frac{\underline{Z}_{10}\underline{Z}_p + \frac{\underline{a}_{12}}{\underline{a}_{21}}\frac{\underline{a}_{22}}{\underline{a}_{22}}}{\underline{Z}_p + \frac{\underline{a}_{22}}{\underline{a}_{21}}} = \frac{\underline{Z}_{10}\underline{Z}_p + \frac{\underline{a}_{12}}{\underline{a}_{22}}\frac{\underline{a}_{22}}{\underline{a}_{21}}}{\underline{Z}_p + \underline{Z}_T} = \frac{\underline{Z}_{10}\underline{Z}_p + \underline{Z}_{1k}\underline{Z}_T}{\underline{Z}_p + \underline{Z}_T}.$$

3. Ako se na krajeve jedne simetrične mreže sa dva para krajeva koja se sastoji samo od otpornika priključi konstantan napon U_1=100 V, struje na pristupima pri kratko spojenim krajevima 2-2' su I_{1k}=8mA i I_{2k}=6mA. Odrediti struje na krajevima kola ako je između krajeva 2-2' vezan otpornik R=10kΩ.



R

Ako se napišu jednačine prema „a“ parametrima:

$$\underline{U}_1 = \underline{a}_{11}\underline{U}_2 + \underline{a}_{12}\underline{I}_2$$
$$\underline{I}_1 = \underline{a}_{21}\underline{U}_2 + \underline{a}_{22}\underline{I}_2 \quad (\#)$$

Uzimajući u obzir da je mreža sastavljena od otpornika, sistem jednačina se ne mora pisati u kompleksnoj formi. Takođe, kako bi se odredila tražena struja potrebno je prethodno odrediti parametre.
Kako su date struje na pristupima pri kratko spojenim krajevima 2-2’ (U_2=0), tada su jednačine:

$$U_1 = a_{12}I_{2k} \rightarrow a_{12} = \frac{U_1}{I_{2k}} = \frac{100}{6} = \frac{50}{3}k\Omega$$

$$I_{1k} = a_{22}I_{2k} \rightarrow a_{22} = \frac{I_{1k}}{I_{2k}} = \frac{8}{6} = \frac{4}{3}$$

Kako je u zadatku navedeno da je mreža simetrična, to važi

$$a_{11} = a_{22} = \frac{4}{3}$$

Takođe, poznato je da važi

$$\det \underset{\sim}{a} = 1 \rightarrow \underset{\sim}{a}_{11}{}^2 - a_{12}a_{21} = 1 \rightarrow a_{21} = \frac{a_{11}{}^2 - 1}{a_{12}} = \frac{7}{150}mS$$

Sada, kada su poznati „a“ parametri moguće je odrediti nepoznatu struju uvrštavajući u sistem (#) uslove koji važe na krajevima,

$$U_1 = 100V$$

$$U_2 = RI_2$$

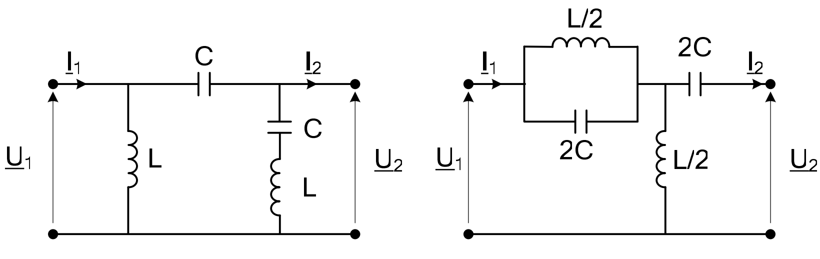
Kada se prethodni uslovi uvrste, dobija se

$$U_1 = a_{11}U_2 + a_{12}I_2$$
$$\underline{I_1 = a_{21}U_2 + a_{22}I_2}$$

$$U_1 = a_{11}RI_2 + a_{12}I_2 \rightarrow I_2 = \frac{U_1}{a_{11}R + a_{12}} = \frac{10}{3}mA$$

$$I_1 = a_{21}RI_2 + a_{22}I_2 = \left(a_{21}R + a_{22}\right)I_2 = 6mA$$

4. Pokazati da su reaktivne mreže sa dva para krajeva prikazane na slici ekvivalentne.



R

Ako se posmatraju „y“ parametri,

$$\begin{bmatrix} \underline{I}_1 \\ -\underline{I}_2 \end{bmatrix} = \begin{bmatrix} \underline{y}_{11} & \underline{y}_{12} \\ \underline{y}_{21} & \underline{y}_{22} \end{bmatrix} \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix}$$

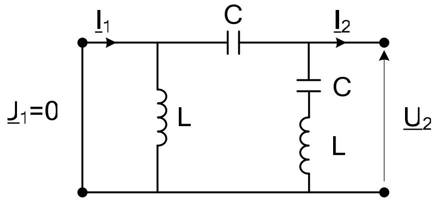
Ekvivalentnost posmatranih mreža se može dokazati ako se pokaže da su „y“ parametri jednaki. Ako se posmatra prva šema

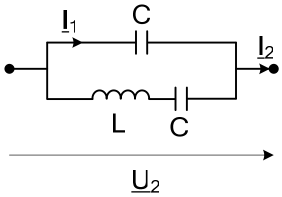
$$\underline{I}_1 = \underline{y}_{11}\underline{U}_1 + \underline{y}_{12}\underline{U}_2$$
$$-\underline{I}_2 = \underline{y}_{21}\underline{U}_1 + \underline{y}_{22}\underline{U}_2$$

$$\underline{y}_{11} = \left.\frac{\underline{I}_1}{\underline{U}_1}\right|_{\underline{U}_2=0} = j\omega C + \frac{1}{j\omega L}$$

$$\underline{y}_{22} = -\left.\frac{\underline{I}_2}{\underline{U}_2}\right|_{\underline{U}_1=0} = j\omega C + \frac{j\omega C\dfrac{1}{j\omega L}}{j\omega C + \dfrac{1}{j\omega L}} = j\omega C + j\omega C\frac{1}{1-\omega^2 LC} = j\omega C\frac{2-\omega^2 LC}{1-\omega^2 LC}$$

$$\underline{y}_{12} = \left.\frac{\underline{I}_1}{\underline{U}_2}\right|_{\underline{U}_1=0} = -jwC = \underline{y}_{21}$$



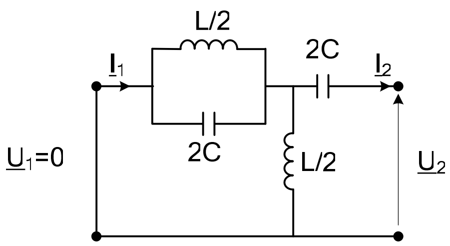


Ako se sad parametri odrede za drugu šemu iz zadatka,

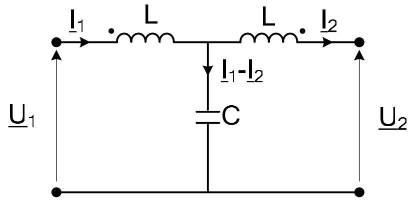
$$\underline{y}_{11} = \left.\frac{\underline{I}_1}{\underline{U}_1}\right|_{\underline{U}_2=0} = \frac{1}{2}\left(j\omega 2C + \frac{2}{j\omega L}\right) = j\omega C + \frac{1}{j\omega L}$$

$$\underline{y}_{22} = -\left.\frac{\underline{I}_2}{\underline{U}_2}\right|_{\underline{U}_1=0} = \frac{\left(j\omega 2C + \dfrac{2}{j\omega L} + \dfrac{2}{j\omega L}\right)j\omega 2C}{j\omega 2C + \dfrac{2}{j\omega L} + \dfrac{2}{j\omega L} + j\omega 2C} = j\omega C\frac{4\dfrac{2-\omega^2 LC}{j\omega L}}{4\dfrac{1-\omega^2 LC}{j\omega L}} = j\omega C\frac{2-\omega^2 LC}{1-\omega^2 LC}$$

$$\underline{y}_{12} = \left.\frac{\underline{I}_1}{\underline{U}_2}\right|_{\underline{U}_1=0} = \frac{\underline{I}_1}{-\left[\dfrac{j\omega\dfrac{L}{2}\dfrac{1}{j2\omega C}}{j\omega\dfrac{L}{2}\dfrac{1}{j2\omega C}} + \dfrac{1}{j2\omega C}\dfrac{j\omega\dfrac{L}{2} + \dfrac{j\omega L}{1-\omega^2 LC}}{j\omega\dfrac{L}{2}}\right]\underline{I}_1} = -jwC = \underline{y}_{21}$$



5. Data je reaktivna mreža sa dva para krajeva.
a) Odrediti impedanse ekvivalentne „T“ šeme i nacrtati njenu šemu
b) Na šta se svodi ekvivalentna mreža u dijelu zadatka pod a) u slučaju kada je koeficijent sprege između kalemova k=1?



R

Potrebno je najprije odrediti parametre mreže sa dva para krajeva. Ako se posmatraju „a“ parametri.

$$\underline{U}_1 = \underline{a}_{11}\underline{U}_2 + \underline{a}_{12}\underline{I}_2$$
$$\underline{I}_1 = \underline{a}_{21}\underline{U}_2 + \underline{a}_{22}\underline{I}_2$$

Ako se posmatra šema, može se napisati

$$\begin{aligned} \underline{U}_1 &= j\omega L\underline{I}_1 - j\omega kL\underline{I}_2 + \frac{1}{j\omega C}\left(\underline{I}_1 - \underline{I}_2\right) \\ \underline{U}_2 &= -j\omega L\underline{I}_2 + \frac{1}{j\omega C}\left(\underline{I}_1 - \underline{I}_2\right) + j\omega kL\underline{I}_1 \end{aligned} \qquad (\#)$$

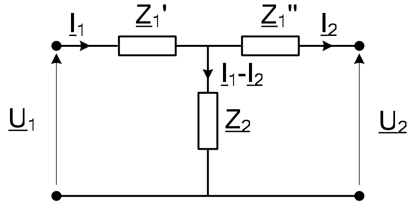
Iz druge jednačine sistema (#) dobija se

$$\underline{U}_2 + j\omega L\underline{I}_2 + \frac{1}{j\omega C}\underline{I}_2 = \left(\frac{1}{j\omega C} + j\omega kL\right)\underline{I}_1 \rightarrow \underline{I}_1 = \underbrace{\frac{1}{j\omega kL + \dfrac{1}{j\omega C}}}_{\underline{a}_{21}}\underline{U}_2 + \underbrace{\frac{1 - w^2LC}{1 - \omega^2 kLC}}_{\underline{a}_{22}}\underline{I}_2$$

Zamjenom u prvu jednačinu sistema (#) dolazi se do

$$\underline{U}_1 = \underbrace{\frac{1 - w^2LC}{1 - \omega^2 kLC}}_{\underline{a}_{11}}\underline{U}_2 + \underbrace{\frac{1}{j\omega C}\frac{\omega^2 LC(k-1)\left(2-(k+1)\omega^2 LC\right)}{1 - \omega^2 kLC}}_{\underline{a}_{12}}\underline{I}_2$$

Iz jednakosti $\underline{a}_{11} = \underline{a}_{22}$ zaključuje se da je mreža simetrična. Ako se nacrta proizvoljna „T“ šema



i napišu jednačine za napon i struju na početku:

$$\underline{U}_1 = \underline{Z}_1{}'\underline{I}_1 + \underline{Z}_1{}''\underline{I}_2 + \underline{U}_2$$

$$\underline{I}_1 = \frac{\underline{U}_2 + \underline{Z}_1{}''\underline{I}_2}{\underline{Z}_2} + \underline{I}_2$$

Zamjenom druge jednačine, prethodnog sistema jednačina, u prvu, dobija se:

$$\underline{U}_1 = \left(1 + \frac{\underline{Z}_1{}'}{\underline{Z}_2}\right)\underline{U}_2 + \left(\underline{Z}_1{}' + \underline{Z}_1{}'' + \frac{\underline{Z}_1{}'\underline{Z}_1{}''}{\underline{Z}_2}\right)\underline{I}_2$$

$$\underline{I}_1 = \frac{1}{\underline{Z}_2}\underline{U}_2 + \left(1 + \frac{\underline{Z}_1{}''}{\underline{Z}_2}\right)\underline{I}_2$$

Data mreža je simetrična što znači da se može realizovati pomoću simetrične „T“ mreže, za koju važi

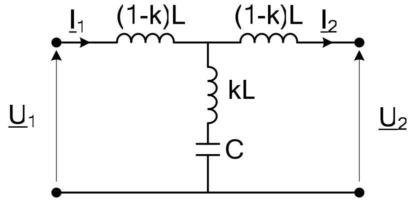
$\underline{Z}_1' = \underline{Z}_1''$

Upoređivanjem sistema jednačina „a“ parametara jedne i druge mreže dolazi se do

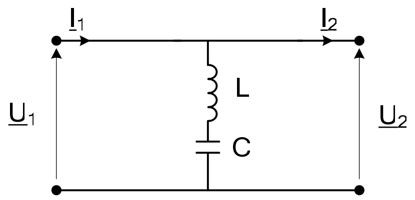
$$\underline{a}_{21} = \frac{1}{j\omega kL + \frac{1}{j\omega C}} = \frac{1}{\underline{Z}_2} \to \underline{Z}_2 = j\omega kL + \frac{1}{j\omega C}$$

$$\underline{a}_{22} = \frac{1-\omega^2 LC}{1-\omega^2 kLC} = 1 + \frac{\underline{Z}_1''}{\underline{Z}_2} \to \underline{Z}_1' = \underline{Z}_1'' = \left(\frac{1-\omega^2 LC}{1-\omega^2 kLC} - 1\right)\underline{Z}_2 = j\omega L\left(1-k\right)$$

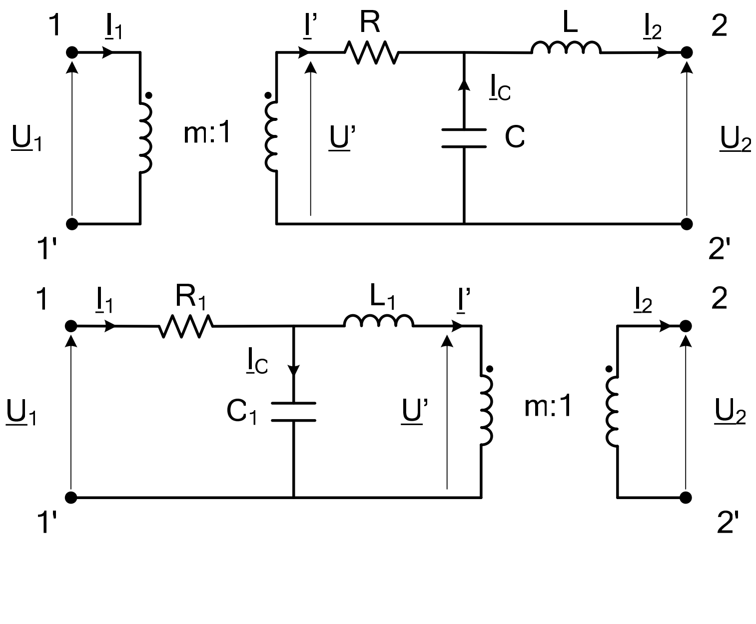
Dakle, ekvivalentna „T“ šema ima oblik



b) u slucaju kada je koeficijent sprege kalemova jednak 1, mreža prelazi u



6.Koji odnos mora postojati između parametara L i L_1, C i C_1, R i R_1 da bi mreže sa dva para krajeva prema slici bile ekvivalentne.



R

Ako se napiše sistem jednačina pomoću „z“ parametara

$$\underline{U}_1 = \underline{z}_{11}\underline{I}_1 + \underline{z}_{12}\left(-\underline{I}_2\right)$$
$$\underline{U}_2 = \underline{z}_{21}\underline{I}_1 + \underline{z}_{22}\left(-\underline{I}_2\right).$$

Za prvu mrežu važi

$$\frac{\underline{U}_1}{\underline{U}'}=m \qquad \frac{\underline{I}_1}{\underline{I}'}=\frac{1}{m} \qquad \underline{I}_C=\underline{I}_2-\underline{I}' \;\rightarrow\; \underline{U}'=\frac{1}{m}\underline{U}_1 \qquad \underline{I}'=m\underline{I}_1 \qquad \underline{I}_C=\underline{I}_2-m\underline{I}_1$$

tada slijedi

$$\underline{U}'=R\underline{I}'-\frac{1}{j\omega C}\underline{I}_C=Rm\underline{I}_1-\frac{1}{j\omega C}\left(\underline{I}_2-m\underline{I}_1\right)=\left(Rm+\frac{m}{j\omega C}\right)\underline{I}_1+\frac{1}{j\omega C}\left(-\underline{I}_2\right)$$

$$\frac{1}{m}\underline{U}_1=\left(Rm+\frac{m}{j\omega C}\right)\underline{I}_1+\frac{1}{j\omega C}\left(-\underline{I}_2\right)\;/\cdot m$$

$$\underline{U}_1=\left(Rm^2+\frac{m^2}{j\omega C}\right)\underline{I}_1+\frac{m}{j\omega C}\left(-\underline{I}_2\right)\rightarrow \underline{z}_{11}=Rm^2+\frac{m^2}{j\omega C} \qquad \underline{z}_{12}=\frac{m}{j\omega C}$$

a napon na krajevima 2-2’ je

$$\underline{U}_2=-j\omega L\underline{I}_2-\frac{1}{j\omega C}\underline{I}_C=-j\omega L\underline{I}_2-\frac{1}{j\omega C}\left(\underline{I}_2-m\underline{I}_1\right)$$

$$\underline{U}_2=\frac{m}{j\omega C}\underline{I}_1+\left(j\omega L+\frac{1}{j\omega C}\right)\left(-\underline{I}_2\right)\rightarrow \qquad \underline{z}_{21}=\frac{m}{j\omega C} \qquad \underline{z}_{22}=j\omega L+\frac{1}{j\omega C}.$$

Ako se posmatra druga mreža iz zadatka, važi

$$\frac{\underline{U}'}{\underline{U}_2}=m \qquad \frac{\underline{I}'}{\underline{I}_2}=\frac{1}{m} \qquad \underline{I}_C=\underline{I}_1-\underline{I}' \;\rightarrow\; \underline{U}'=m\underline{U}_2 \qquad \underline{I}'=\frac{1}{m}\underline{I}_2 \qquad \underline{I}_C=\underline{I}_1-\frac{1}{m}\underline{I}_2$$

tada je napon

$$\underline{U}_1=R_1\underline{I}_1+\frac{1}{j\omega C_1}\underline{I}_C=R_1\underline{I}_1+\frac{1}{j\omega C_1}\left(\underline{I}_1-\frac{1}{m}\underline{I}_2\right)$$

$$\underline{U}_1=\left(R_1+\frac{1}{j\omega C_1}\right)\underline{I}_1+\frac{1}{j\omega mC_1}\left(-\underline{I}_2\right)\rightarrow \qquad \underline{z}_{11}=R_1+\frac{1}{j\omega C_1} \qquad \underline{z}_{12}=\frac{1}{j\omega mC_1}.$$

Napon sa druge strane je

$$\underline{U}'=-j\omega L_1\underline{I}'+\frac{1}{j\omega C_1}\underline{I}_C$$

$$m\underline{U}_2=\frac{1}{j\omega C_1}\underline{I}_1+\left(\frac{j\omega L_1}{m}+\frac{1}{j\omega C_1 m}\right)\left(-\underline{I}_2\right)\;/\cdot\frac{1}{m}$$

$$\underline{U}_2=\frac{1}{j\omega mC_1}\underline{I}_1+\left(\frac{j\omega L_1}{m^2}+\frac{1}{j\omega m^2C_1}\right)\left(-\underline{I}_2\right)\rightarrow \qquad \underline{z}_{21}=\frac{1}{j\omega mC_1} \qquad \underline{z}_{22}=\frac{j\omega L_1}{m^2}+\frac{1}{j\omega m^2C_1}.$$

Da bi mreže bile ekvivalentne parametri moraju biti jednaki, pa važi

$$\underline{z}_{11}=\underline{z}_{11}\rightarrow Rm^2+\frac{m^2}{j\omega C}=R_1+\frac{1}{j\omega C_1}\rightarrow Rm^2=R_1\rightarrow\frac{R_1}{R}=m^2 \qquad \frac{m^2}{j\omega C}=\frac{1}{j\omega C_1}\rightarrow\frac{C}{C_1}=m^2$$

$$\underline{z}_{22}=\underline{z}_{22}\rightarrow\frac{1-\omega^2LC}{j\omega C}=\frac{1-\omega^2L_1C_1}{jm^2\omega C_1}\rightarrow\frac{L_1}{L}=m^2.$$

Dakle, traženi odnosi su

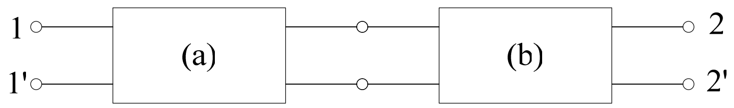
$$\frac{R_1}{R}=\frac{L_1}{L}=\frac{C}{C_1}=m^2$$

7. Dvije mreže sa dva para krajeva vezane su kaskadno. Njihove h matrice su:

$$\underset{\sim}{h}^{(a)}=\begin{bmatrix} s & -3 \\ -s & s+3 \end{bmatrix}, \underset{\sim}{h}^{(b)}=\begin{bmatrix} 4 & 1 \\ -1 & s+1 \end{bmatrix}$$

a) odrediti $\underline{h}$ matricu rezultujuće mreže
b) odrediti ekvivalentnu T mrežu.

Rješenje:



a) Za kaskadnu vezu najpovoljniji su a parametri, jer važi:

$$\underset{\sim}{a}=\underset{\sim}{a}^{(a)}\underset{\sim}{a}^{(b)}$$

Važi da je:

$$\underset{\sim}{a}^{(a)}=\frac{\begin{bmatrix} -\det\left\{\underset{\sim}{h}^{(a)}\right\} & -h_{11}^{(a)} \\ -h_{22}^{(a)} & -1 \end{bmatrix}}{h_{21}^{(a)}}=\begin{bmatrix} s & 1 \\ \frac{s+3}{s} & \frac{1}{s} \end{bmatrix}$$

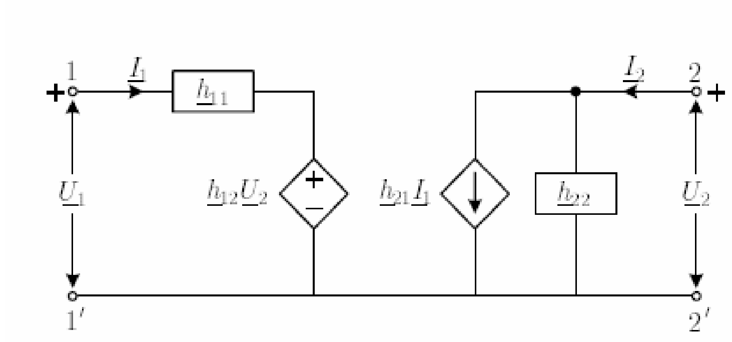
$$\underset{\sim}{a}^{(b)}=\begin{bmatrix} 4s+5 & 4 \\ s+1 & 1 \end{bmatrix}$$

$$\underset{\sim}{a}=\underset{\sim}{a}^{(a)}\underset{\sim}{a}^{(b)}=\begin{bmatrix} s & 1 \\ \frac{s+3}{s} & \frac{1}{s} \end{bmatrix}\begin{bmatrix} 4s+5 & 4 \\ s+1 & 1 \end{bmatrix}=\begin{bmatrix} 4s^2+6s+1 & 4s+1 \\ \frac{4s^2+18s+16}{s} & \frac{4s+13}{s} \end{bmatrix}$$

Sada se na osnovu veze $\underline{a}$ i $\underline{h}$ matrice nalazi:

$$\underset{\sim}{h}=\begin{bmatrix} \frac{a_{12}}{a_{22}} & \frac{\det(\underset{\sim}{a})}{a_{22}} \\ \frac{-1}{a_{22}} & \frac{a_{21}}{a_{22}} \end{bmatrix}=\frac{\begin{bmatrix} 4s+1 & -\frac{3}{s} \\ -1 & \frac{4s^2+18s+16}{s} \end{bmatrix}}{\frac{4s+13}{s}}=\begin{bmatrix} \frac{s(4s+1)}{4s+13} & -\frac{3}{4s+13} \\ \frac{-s}{4s+13} & \frac{4s^2+18s+16}{4s+13} \end{bmatrix}$$

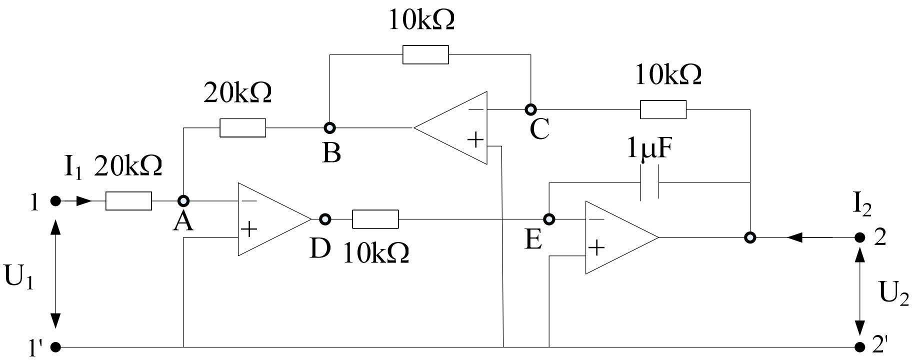
b) Pošto nije ispunjen uslov $h_{12}=-h_{21}$ mreža nije recipročna. Za ekvivalentnu T mrežu nerecipročne mreže preko $\underline{h}$ parametara dobija se šema:



$$\underline{U_1}=h_{11}\underline{I_1}+h_{12}\underline{U_2}$$
$$\underline{I_2}=h_{21}\underline{I_1}+h_{22}\underline{U_2}$$

Vrijednosti impedansi ikontrolisanih izvora su označene na slici.

8. Za kolo na slici odrediti g parametre. Nakon toga odrediti i matrice h, y, z, i a.



Rješenje:

Odredimo napone tačaka označenih na slici:

A: $\left(\frac{1}{20}+\frac{1}{20}\right)V_A-\frac{1}{20}V_B-\frac{1}{20}V_1=0$

C: $\left(\frac{1}{10}+\frac{1}{10}\right)V_C-\frac{1}{10}V_B-\frac{1}{10}V_2=0$

E: $\left(\frac{1}{10}+s\cdot 10^{-3}\right)V_E-\frac{1}{10}V_D-s\cdot 10^{-3}V_2=0$

$V_A=V_E=V_C=0$

Slijedi:

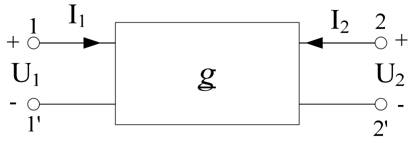
$$\left.\begin{aligned}-V_1-V_B=0\\-V_B-V_2=0\end{aligned}\right\}\Rightarrow V_1=V_2$$

$-V_D-10^{-2}sV_2=0$

Treba još jedna jednačina.

$V_1=20\cdot 10^3\cdot I_1+V_A\Rightarrow I_1=\frac{V_1}{20\cdot 10^3}$

g parametri su opisani sledećim relacijama:



$$\begin{bmatrix}\underline{I_1}\\\underline{U_2}\end{bmatrix}=\begin{bmatrix}g_{11} & g_{12}\\g_{21} & g_{22}\end{bmatrix}\begin{bmatrix}U_1\\I_2\end{bmatrix}$$

Tj: $\begin{aligned}\underline{I_1}&=g_{11}\underline{U_1}+g_{12}\underline{I_2}\\\underline{U_2}&=g_{21}\underline{U_1}+g_{22}\underline{I_2}\end{aligned}$

Iz gornjih relacija važe jednačine:

$\underset{\sim}{h}=\underset{\sim}{g}^{-1}\ I_1=\frac{1}{20\cdot 10^3}V_1+0\underline{I_2}\Rightarrow\boxed{g_{11}=\frac{1}{20\cdot 10^3}}$ i $\boxed{g_{12}=0}$

$\underline{V_2}=\underline{V_1}+0\underline{I_2}\Rightarrow\boxed{g_{21}=1}$ i $\boxed{g_{22}=0}$

Prema tome:

$$\underset{\sim}{g}=\begin{bmatrix}1/2\cdot 10^4 & 0\\1 & 0\end{bmatrix},\quad \det\left(\underset{\sim}{g}\right)=0$$

Kako je $\det\left(\underset{\sim}{g}\right)=0$, a $\underset{\sim}{h}=\underset{\sim}{g}^{-1}$, matrica *h* ne egzistira.

$$\underset{\sim}{y}=\begin{bmatrix}\frac{\det\left\{\underset{\sim}{g}\right\}}{g_{22}} & \frac{g_{12}}{g_{22}}\\ -\frac{g_{21}}{g_{22}} & \frac{1}{g_{22}}\end{bmatrix},\quad g_{22}=0$$, pa ni matrica *y* ne egzistira.

$$\underset{\sim}{a}=\begin{bmatrix}\frac{1}{g_{21}} & \frac{g_{22}}{g_{21}}\\ \frac{g_{11}}{g_{21}} & \frac{\det\left\{\underset{\sim}{g}\right\}}{g_{21}}\end{bmatrix}=\begin{bmatrix}1 & 0\\ \frac{1}{2\cdot 10^{4}} & 0\end{bmatrix}$$

$$\underset{\sim}{z}=\begin{bmatrix}\frac{1}{g_{11}} & -\frac{g_{12}}{g_{11}}\\ \frac{g_{21}}{g_{11}} & \frac{\det\left\{\underset{\sim}{g}\right\}}{g_{11}}\end{bmatrix}=\begin{bmatrix}2\cdot 10^{4} & 0\\ 2\cdot 10^{4} & 0\end{bmatrix}$$