## ECTS catalog with learning outcomes University of Montenegro

Faculty of Science and Mathematics / MATHEMATICS / LINEAR ALGEBRA 1

| Course: | LINEAR ALGEBRA 1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Course ID | Course status | Semester | ECTS credits | Lessons (Lessons+Exer cises+Laboratory) |
| 3967 | Mandatory | 1 | 8 | $4+3+0$ |
| Programs | MATHEMATICS |  |  |  |
| Prerequisites | no |  |  |  |
| Aims | Standard course of Linear algebra for students of mathematics. Includes theory of finite-dimensional vector spaces, matrices, systems of linear equations and linear mappings in finite-dimensional vector spaces (including spectral theory). |  |  |  |
| Learning outcomes |  |  |  |  |
| Lecturer / Teaching assistant | Vladimir Jaćimović, Dušica Slović |  |  |  |
| Methodology | lectures, seminars, consultations |  |  |  |
| Plan and program of work |  |  |  |  |
| Preparing week | Preparation and registration of the semester |  |  |  |
| I week lectures | Groups and fields. Vector spaces. Definition. Examples. Vector subspaces. Linear span. |  |  |  |
| I week exercises | Groups and fields. Fields of real and complex numbers. Geometric vectors in the plane. |  |  |  |
| II week lectures | Linearly dependent and independent vectors. Base and dimension of vector spaces. Isomorfism of vector spaces. |  |  |  |
| II week exercises | Vector spaces. $\mathrm{R}^{\wedge} \mathrm{n}$ and $\mathrm{C}^{\wedge} \mathrm{n}$. Vector subspaces. Linear span. |  |  |  |
| III week lectures | Matrices. Gauss method for solving linear systems of equations. Matrices of elementary transforms. |  |  |  |
| III week exercises | Linearly dependent and independent vectors. Base and dimension of vector spaces. Problems and examples in $\mathrm{R}^{\wedge} \mathrm{n}$. Subspaces in $\mathrm{R}^{\wedge} \mathrm{n}$. Systems of linear equations. |  |  |  |
| IV week lectures | Determinants of square matrices. Rank of matrix. |  |  |  |
| IV week exercises | Gauss method for solving systems of linear equations. Matrices. Matrices of elementary transforms. |  |  |  |
| V week lectures | Inverse matrix. Regular and singular matrices. Matrices of change of bases. Equivalent matrices. |  |  |  |
| V week exercises | Determinant and rank of matrix. |  |  |  |
| VI week lectures | Systems of linear equations. Existence and uniqueness of solution. General solution. Kronecker Capelli theorem. Cramers' rule. |  |  |  |
| VI week exercises | Inverse matrix. Regular and singular matrices. Matrices of coordinate change. |  |  |  |
| VII week lectures | 1st test |  |  |  |
| VII week exercises | 1st test |  |  |  |
| VIII week lectures | Empty week. |  |  |  |
| VIII week exercises | Empty week. |  |  |  |
| IX week lectures | Linear mappings in vector spaces. Definition. Examples. Kernel and image of linear mapping. |  |  |  |
| IX week exercises | Homogeneous and nonhomogeneous systems of linear equations. Methods of solving. Existence and uniqueness of solution. Cramers' rule. |  |  |  |
| X week lectures | Matrix of linear mapping. Similar matrices. Inverse mapping. Rank of linear mapping. |  |  |  |
| X week exercises | Linear mappings in vector spaces. Kernel and image of linear mapping. Examples: operators of projection, rotation and differentiation of polynomials. |  |  |  |
| XI week lectures | Invariant subspaces of linear mapping. Eigenvalues and eigenvectors. Eigenspaces. |  |  |  |
| XI week exercises | Matrix of linear mapping. Inverse mapping. Rank of linear mapping. |  |  |  |
| XII week lectures | Fundamental theorem of algebra. Characteristic polynomial of linear mapping. Polynomials of matrices/operators. Hamilton-Cayley theorem. |  |  |  |
| XII week exercises | Eigenvalues and eigenvectors of linear mapping. Characteristic polynomial of linear mapping. |  |  |  |
| XIII week lectures | Jordan form and cannonical base of nilpotent linear mapping. |  |  |  |
| XIII week exercises | Method of calculation of eigenvectors. Eigenspaces. |  |  |  |


| XIV week lectures |  | Jordan form of linear mapping. Examples. |  |  |  |  |
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| XIV week exercises |  | Jordan form of linear mapping. Similar matrices. |  |  |  |  |
| XV week lectures |  | 2nd test |  |  |  |  |
| XV week exercises |  | 2nd test |  |  |  |  |
| Student workload |  | ```4 hours/week lectures + 3 hours/week seminars + 4 hours/week homework = 11 hours/week. Total: 11 hours/week x 16 weeks = 176 hours``` |  |  |  |  |
| Per week |  |  | Per semester |  |  |  |
| $\mathbf{8}$ credits $\mathbf{x} \mathbf{4 0} / \mathbf{3 0}=\mathbf{1 0}$ hours and $\mathbf{4 0}$ minuts <br> 4 sat(a) theoretical classes <br> 0 sat(a) practical classes <br> 3 excercises <br> 3 hour(s) i 40 minuts <br> of independent work, including consultations |  |  | Classes and final exam: <br> $\mathbf{1 0}$ hour(s) i $\mathbf{4 0}$ minuts $\mathbf{x} \mathbf{1 6}=\mathbf{1 7 0}$ hour(s) i $\mathbf{4 0}$ minuts <br> Necessary preparation before the beginning of the semester (administration, registration, certification): <br> $\mathbf{1 0}$ hour(s) i $\mathbf{4 0}$ minuts $\mathbf{x} \mathbf{2} \mathbf{= 2 1}$ hour(s) i $\mathbf{2 0}$ minuts <br> Total workload for the subject: <br> $\mathbf{8 \times 3 0 = 2 4 0}$ hour(s) <br> Additional work for exam preparation in the preparing exam period, including taking the remedial exam from 0 to 30 hours (remaining time from the first two items to the total load for the item) <br> 48 hour(s) i 0 minuts <br> Workload structure: $\mathbf{1 7 0}$ hour(s) i 40 minuts (cources), 21 hour(s) i 20 minuts (preparation), 48 hour(s) i 0 minuts (additional work) |  |  |  |
| Student obligations |  |  |  |  |  |  |
| Consultations |  |  | 1 hour/week (lectures) + 1 hour/week (seminars) |  |  |  |
| Literature |  |  | M. Jaćimović, I. Krnić „Linearna algebra, teoreme i zadaci" (skripta) E. Shikin "Lineinie prostranstva i otobrazheniya", Moskva 1987. S. Friedberg, A. Insel, L. Spence „Linear algebra, 4th edition" Pearson, 2002. |  |  |  |
| Examination methods |  |  | attendance ( 5 points), homework ( $5 \times 1$ points), 2 tests ( $2 \times 30$ points), one corrective test, final exam ( 30 points), corrective final exam, 2 brief oral exams (optional $-2 \times 5$ points) |  |  |  |
| Special remarks |  |  | The language of instruction is Serbo-Croat. Lectures can be given in English or Russian language. |  |  |  |
| Comment |  |  |  |  |  |  |
| Grade: | F | E | D | C | B | A |
| Number of points | less than 50 points | greater than or equal to 50 points and less than 60 points | greater than or equal to 60 points and less than 70 points | greater than or equal to 70 points and less than 80 points | greater than or equal to 80 points and less than 90 points | greater than or equal to 90 points |

